The number of distinct real roots of the equation
$x^5(x^3-x^2-x+1)+x(3x^3-4x^2-2x+4)-1=0$ is _____.

Solution

We have, $x^5(x^3-x^2-x+1)+x(3x^3-4x^2-2x+4)-1=0$

$\therefore x^5\{x^2(x-1)-(x-1)\}+(3x^4-4x^3-2x^2+4x-1)=0$

$\therefore x^5(x-1)(x^2-1)+\{3x^3(x-1)-x^2(x-1)-3x(x-1)+(x-1)\}=0$

$\therefore x^5(x-1)^2(x+1)+(x-1)(3x^3-x^2-3x+1)=0$

$\therefore x^5(x-1)^2(x+1)+(x-1)\{3x^2(x+1)-4x(x+1)+(x+1)\}=0$

$\therefore x^5(x-1)^2(x+1)+(x-1)(x+1)(3x^2-4x+1)=0$

$\therefore x^5(x-1)^2(x+1)+(x-1)(x+1)(3x^2-3x-x+1)=0$

$\therefore x^5(x-1)^2(x+1)+(x-1)(x+1)(x-1)(3x-1)=0$

$\Rightarrow (x-1)^2(x+1)(x^5+3x-1)=0$

Let $f(x)=x^5+3x-1$

$f(-\infty)\to-\infty$ and $f(+\infty)\to+\infty$

$f'(x)=5x^4+3>0$

So, f(x) is strictly increasing cutting x-axis at only 1 point.

Since $f(-1)\neq 0$ and $f(1)\neq 0$, the point where f(x) cuts axis is different from -1 and 1.

Hence, 3 distinct roots.