

$$f(x) = \int_0^x \pi^t (t - e)(t - \pi) dt$$

Find interval(s) in which f increases.

Solution

Using the formula $\frac{d}{dx} \left[\int_a^x g(x, u) du \right] = g(x, x) + \int_a^x \frac{\partial}{\partial x} g(x, u) du$, we have

$$f'(x) = \pi^x (x - e)(x - \pi)$$

Let us look at the sign scheme for $f'(x)$.



For increasing function, we have $f'(x) \geq 0$

So, f increases in the intervals $(-\infty, e] \cup [\pi, \infty)$.