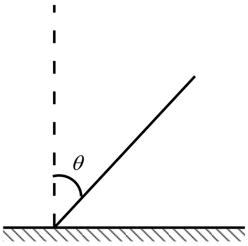
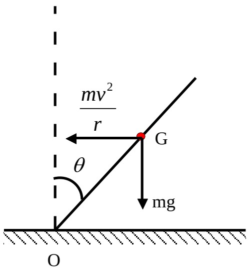
A stuntman riding a motorcycle wishes to take a turn of radius 100 m at a speed of 108 kmph. At what angle should he bend from the vertical for the safe ride (see fig.)? Assume that there is no skidding and the only point of concern is toppling. (g = 10 m/s^2, $\tan 3^\circ = 0.0524$)



Solution



In the figure above, the forces passing through O are not shown as torque about O is not produced by those forces.

Considering rotational equilibrium about O, $\tau_{mg} = \tau_{\frac{mv^2}{r}}$

Here, $\frac{mv^2}{r}$ is the centrifugal force with respect to the motorcyclist observer. The torque due to this centrifugal force tries to push him away from the circular path. Torque due to mg is needed to counter this torque. That's the reason why the motorcyclist bends.

$$\therefore mg.OG\sin\theta = \frac{mv^2}{r}.OG\cos\theta$$

G is the centre of gravity.

$$\Rightarrow \tan\theta = \frac{v^2}{rg} = \frac{30^2}{100\times 10} = 0.9$$

Clearly, the angle is a bit smaller than 45°.

Since $\tan 3^\circ$ is given let's try $\tan 45^\circ = 1 = \tan(42^\circ + 3^\circ) = \frac{\tan 42^\circ + \tan 3^\circ}{1 - \tan 42^\circ \tan 3^\circ} = \frac{x + \tan 3^\circ}{1 - x\tan 3^\circ}$

$$\therefore 1 - x\tan 3^\circ = x + \tan 3^\circ$$

$$\Rightarrow x = \frac{1-\tan 3^\circ}{1+\tan 3^\circ} = \frac{1-0.0524}{1+0.0524} = \frac{0.9476}{1.0524} \approx 0.9$$

$$\therefore \theta = 42^\circ$$

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