A gun is mounted on a gun carriage movable on a smooth horizontal plane. The gun is elevated at an angle α to the horizontal. A shot is fired which leaves the gun in a direction inclined at an angle β to the horizontal. If the mass of gun+carriage is n times that of the shot, show that $\tan\beta = \left(1+\frac{1}{n}\right)\tan\alpha$.

Figure (I): As seen by the shooter.

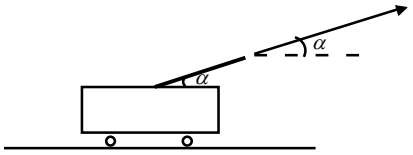
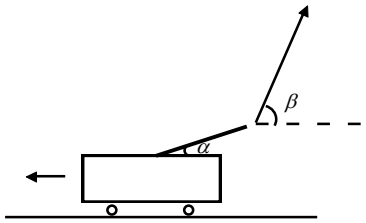
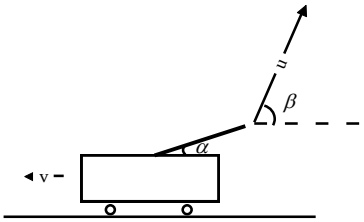


Figure (II): As seen by the ground observer.



Solution

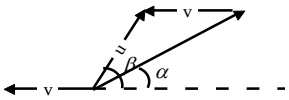


Consider ground observer. Let v be the recoil speed and u be the speed of the bullet. Let m be the mass of the bullet. So, the mass of carriage+gun system would be nm.

Using COLM along horizontal, $m.u\cos\beta = nm.v$

$$\therefore v = \frac{u\cos\beta}{n}$$

Now, consider the shooter as the observer.



In the figure above, the vector at angle α to the horizontal denotes the velocity of bullet with respect to the shooter.

We have, $\tan\alpha = \frac{u\sin\beta}{u\cos\beta+v} = \frac{u\sin\beta}{u\cos\beta+\frac{u\cos\beta}{n}} = \frac{\tan\beta}{1+\frac{1}{n}}$

$$\therefore \tan\beta = \left(1+\frac{1}{n}\right)\tan\alpha$$

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