$$7f(x)+5f\left(\frac{1}{x}\right)=x+1$$

$$t=24xf(x)$$

$$\left.\frac{dt}{dx}\right|_{x=-1/7}=?$$

Solution

We have, $7f(x)+5f\left(\frac{1}{x}\right)=x+1$.....(A)

Replacing x by $\frac{1}{x}$, $7f\left(\frac{1}{x}\right)+5f(x)=\frac{1}{x}+1$ .....(B)

(A) + (B) yields, $12\left[f(x)+f\left(\frac{1}{x}\right)\right]=x+\frac{1}{x}+2$

$\therefore f(x)+f\left(\frac{1}{x}\right)=\frac{x}{12}+\frac{1}{12x}+\frac{1}{6}$.....(C)

(A) - (B) yields, $2\left[f(x)-f\left(\frac{1}{x}\right)\right]=x-\frac{1}{x}$

$\therefore f(x)-f\left(\frac{1}{x}\right)=\frac{x}{2}-\frac{1}{2x}$.....(D)

(C) + (D) yields, $2f(x)=\frac{7x}{12}-\frac{5}{12x}+\frac{1}{6}$

$\therefore f(x)=\frac{7x}{24}-\frac{5}{24x}+\frac{1}{12}$

Now, $t=24xf(x)=7x^2-5+2x$

$\therefore \left.\frac{dt}{dx}\right|_{x=-1/7}=14x+2\big|_{x=-1/7}=0$