The number of solution(s) the equation $x^{x^2} = x^{4x+5}$ has:

(A) One solution only   (B) Two solutions
(C) Three solutions   (D) No solution

Answer

When $a^b = a^c$ then b = c if $a \neq 1$

So, we take two cases, (I) $a \neq 1$ (II) a = 1

Case (I)

$x^{x^2} = x^{4x+5}$, $x \neq 1$

Here, $x^2 = 4x + 5$

$\Rightarrow (x-5)(x+1) = 0$

$\Rightarrow x \equiv \{5, -1\}$

Case (II)

$x^{x^2} = x^{4x+5}$, $x = 1$

Clearly, x = 1 is also a solution.

So, there are three solutions in total, $\{5, \pm 1\}$