Solve for x & y; $\sqrt{x}+y=7$ & $x+\sqrt{y}=11$.

Solution

Putting $y=(11-x)^2$ obtained from the 2nd equation into the 1st equation,

We have, $\sqrt{x}+(11-x)^2=7$

$\Rightarrow x=[7-(11-x)^2]^2$

$=49+(11-x)^4-14(11-x)^2$

Let, $11-x=t$

So, $11-t=49+t^4-14t^2$

$\Rightarrow t^4-14t^2+t+38=0$

t=2 satisfies the above equation. So, using factor theorem …

$t^3(t-2)+2t^2(t-2)-10t(t-2)-19(t-2)=0$

$\Rightarrow (t-2)(t^3+2t^2-10t-19)=0$ ……….. (A)

Let, $f(t)=t^3+2t^2-10t-19$

$t=11-x=\sqrt{y}\geq 0$

$y=7-\sqrt{x}, \Rightarrow y\leq 7$

So, $t=\sqrt{y}\leq\sqrt{7}$

Thus, $0\leq t\leq\sqrt{7}$

Or, $0\leq t<3$

Let us investigate the behaviour of f in the interval [0, 3].

f(0) = -19

f(1) = -26

f(2) = -23

f(3) = -4

The function decreases initially, then increases but is unable to cut the t-axis in the interval [0, 3).

t = 2 is the only solution as obtained earlier as per (A).

Now, $11-x=2$ or $x=9$

$y=t^2=4$