The equation of the plane containing the line $2x-5y+z=3$, $x+y+4z=5$ and parallel to the plane $x+3y+6z=1$ is

(1) $x+3y+6z=-7$
(2) $x+3y+6z=7$
(3) $2x+6y+12z=-13$
(4) $2x+6y+12z=13$

Solution

Equation of plane parallel to $x+3y+6z=1$ is $P \equiv x+3y+6z=k$

Plane passing through the intersection of $2x-5y+z=3$ and $x+y+4z=5$ is given by
$P' \equiv (2x-5y+z-3)+\lambda(x+y+4z-5)=0$

Or, $P' \equiv (2+\lambda)x+(-5+\lambda)y+(1+4\lambda)z=3+5\lambda$

$\because P \equiv P'$, $\dfrac{2+\lambda}{1}=\dfrac{-5+\lambda}{3}=\dfrac{1+4\lambda}{6}=\dfrac{3+5\lambda}{k}$

$\dfrac{2+\lambda}{1}=\dfrac{-5+\lambda}{3}$ yields $\lambda=-\dfrac{11}{2}$

$\dfrac{2+\lambda}{1}=\dfrac{3+5\lambda}{k}$ yields $k=7$

Hence, the required plane is $x+3y+6z=7$

Option (2)