If f(a)=2, f'(a)=1, g(a)=-1 and g'(a)=2, the value of

$$\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$$

is

(A) -5 (B) $\frac{1}{5}$ (C) 5 (D) none of these

Solution

The asked limit = $\lim_{x\to a}\frac{g(x)f(a)-g(x)f(x)+g(x)f(x)-g(a)f(x)}{x-a}$

= $\lim_{x\to a}\frac{g(x)\{f(a)-f(x)\}+f(x)\{g(x)-g(a)\}}{x-a}$

=- $\lim_{x\to a} g(x)\frac{f(x)-f(a)}{x-a}+f(x)\frac{g(x)-g(a)}{x-a}$

$=-\lim_{x\to a} g(x)\lim_{x\to a}\frac{f(x)-f(a)}{x-a}+\lim_{x\to a} f(x)\lim_{x\to a}\frac{g(x)-g(a)}{x-a}$

$=-\lim_{x\to a} g(x)f'(a)+\lim_{x\to a} f(x)g'(a)=-\lim_{x\to a} g(x)+2\lim_{x\to a} f(x)$

Since g'(a) exists, g(x) is continuous at a. Thus, $\lim_{x\to a} g(x)=g(a)$

Similarly, $\lim_{x\to a} f(x)=f(a)$

The asked limit = $-g(a)+2f(a)$=$1+2\times 2=5$

Hence, (C)