

Let t_i be the length of tangent from some outside point on the family of circles $x^2 + y^2 + 2g_ix + \lambda = 0$, $i = 1, 2, 3$ then the points (g_i, t_i^2) , $i = 1, 2, 3$ are

- (A) Collinear (B) Non-collinear
(C) May be collinear (D) Nothing can be inferred about collinearity due to unknown λ

Solution

Let the outside point be (u, v) .

$$\text{Then, } t_i^2 = u^2 + v^2 + 2g_i u + \lambda$$

For three points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , the condition for collinearity is,

$$\sum x_1(y_2 - y_3) = 0$$

$$\begin{aligned} \text{Now, } \sum g_1(t_2^2 - t_3^2) &= \sum g_1\{(u^2 + v^2 + 2g_2u + \lambda) - (u^2 + v^2 + 2g_3u + \lambda)\} \\ &= 2u \sum g_1(g_2 - g_3) \text{ which is zero} \end{aligned}$$

Hence, (A)