The period of function $f(x)=\dfrac{\sin 2x}{\sin\dfrac{x}{2}}$ is:

(A) $\dfrac{\pi}{2}$ (B) π (C) 2π (D) 4π

Solution

$$f(x)=\frac{2\sin x\cos x}{\sin\frac{x}{2}}=\frac{4\sin\frac{x}{2}\cos\frac{x}{2}\cos x}{\sin\frac{x}{2}}$$

$$f(x)=4\cos\frac{x}{2}\cos x,\left(\sin\frac{x}{2}\neq 0\right)$$

$$\therefore f(x)=2\left(2\cos\frac{x}{2}\cos x\right)=2\left(\cos\frac{3x}{2}+\cos\frac{x}{2}\right)$$

Period of f(x) is same as the period of $\cos\dfrac{3x}{2}+\cos\dfrac{x}{2}$

Period of $\cos\dfrac{3x}{2}=\dfrac{2\pi}{3/2}=\dfrac{4\pi}{3}$ & Period of $\cos\dfrac{x}{2}=\dfrac{2\pi}{1/2}=4\pi$

Period of f(x) = LCM of $\dfrac{4\pi}{3}$ & 4π = $\dfrac{\text{LCM of }(4\pi,4\pi)}{\text{HCF of }(3,1)}=\dfrac{4\pi}{1}=4\pi$

Answer: (D)