If a, b and c are three positive real numbers, then the minimum value of the expression

$$\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}$$

is

(A) 1 (B) 2 (C) 3 (D) none of these

Solution

The given expression can be written as $\left(\frac{b}{a}+\frac{a}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)+\left(\frac{c}{b}+\frac{b}{c}\right)$

Using $AM \geq GM$, $\frac{\frac{a}{b}+\frac{b}{a}}{2} \geq \sqrt{\frac{a}{b}.\frac{b}{a}}$

$\therefore \frac{a}{b}+\frac{b}{a} \geq 2$

Likewise, $\frac{b}{c}+\frac{c}{b} \geq 2$ & $\frac{c}{a}+\frac{a}{c} \geq 2$

Adding, we have $\left(\frac{b}{a}+\frac{a}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)+\left(\frac{c}{b}+\frac{b}{c}\right) \geq 6$

$\therefore \left(\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}\right)_{\min} = 6$

Answer: (D)