The sum to 50 terms of the series,

$$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+.....$$

is [select correct option(s)]

(A) > 1 (B) < 1 (C) $\frac{50}{51}\times\frac{52}{51}$ (D) $\frac{50}{51}$

Solution

$$t_r=\frac{2r+1}{r^2.(r+1)^2}=\frac{1}{r^2}-\frac{1}{(r+1)^2}$$

$$S_n=\sum t_r=\sum\frac{1}{r^2}-\frac{1}{(r+1)^2}$$

$$S_n=\frac{1}{1^2}-\frac{1}{2^2}$$

$$+\frac{1}{2^2}-\frac{1}{3^2}$$

$$+\frac{1}{3^2}-\frac{1}{4^2}$$

$$+.....$$

$$+\frac{1}{n^2}-\frac{1}{(n+1)^2}$$

$$S_n=\frac{1}{1^2}-\frac{1}{(n+1)^2}=\frac{n^2+2n}{(n+1)^2}=\frac{n(n+2)}{(n+1)^2}$$

$$\therefore S_{50}=\frac{50}{51}\times\frac{52}{51}$$

Also, $S_n=\frac{1}{1^2}-\frac{1}{(n+1)^2}<1$

Hence, (B) & (C)