

The sum of 51 terms of the series

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$$

is equal to

(A) $2^{51} - 52$

(B) $1 - \frac{1}{2^{51}}$

(C) $\frac{1}{2^{51}} + 50$

(D) $2^{51} - 1$

Solution

$$\text{Let } S_n = \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{16}\right) + \dots + (t_n)$$

$$S_n = n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots n \text{ terms}\right)$$

$$= n - \frac{\frac{1}{2} \left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}}$$

$$= n - \left(1 - \frac{1}{2^n}\right) = \frac{1}{2^n} + n - 1$$

$$\therefore S_{51} = \frac{1}{2^{51}} + 50$$

Hence, (C)