In a triangle ABC, $(a+b+c)(b+c-a)=\lambda bc$ then

(A) $\lambda<0$ (B) $\lambda>6$ (C) $0<\lambda<4$ (D) $\lambda>4$

Solution

We have, $(\overline{b+c}+a)(\overline{b+c}-a)=\lambda bc$

$\therefore (b+c)^2-a^2=\lambda bc$

$\therefore b^2+c^2-a^2+2bc=\lambda bc$

$\therefore \dfrac{b^2+c^2-a^2}{bc}+2=\lambda$

$\therefore 2\cos A+2=\lambda$

$\therefore \lambda=2(1+\cos A)=4\cos^2\dfrac{A}{2}$

Now, $0<A<\pi$ OR $0<\dfrac{A}{2}<\dfrac{\pi}{2}$

So, $0<\cos\dfrac{A}{2}<1$ OR $0<\cos^2\dfrac{A}{2}<1$ OR $0<4\cos^2\dfrac{A}{2}<4$

Thus, $0<\lambda<4$

Hence, (C)