If the circles $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$ touch each other then,

$$(2gg'+2ff'-c-c')^2=4(g^2+f^2-c)(g'^2+f'^2-c')$$

is

(A) always true
(B) true only if circles touch externally
(C) true only if circles touch internally
(D) always false

Solution

Distance between centres = $|r\pm r'|$

$\therefore \sqrt{(g-g')^2+(f-f')^2}=|r\pm r'|$

$\therefore (g-g')^2+(f-f')^2=(r\pm r')^2$

$\therefore g^2+g'^2-2gg'+f^2+f'^2-2ff'=r^2+r'^2\pm 2rr'=g^2+f^2-c+g'^2+f'^2-c'\pm 2rr'$

$\therefore -2gg'-2ff'=-c-c'\pm 2rr'$

$\therefore c+c'-2gg'-2ff'=\pm 2rr'$

$\therefore (2gg'+2ff'-c-c')^2=4r^2r'^2=4(g^2+f^2-c)(g'^2+f'^2-c')$

Hence, (A)