Two isolated metallic solid spheres of radius R and 2R are charged such that both of these have same surface charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. The new charge density on the bigger sphere is:

(A) $\frac{5}{6}\sigma$ (B) $\frac{6}{5}\sigma$ (C) $\frac{2}{3}\sigma$ (D) σ

Solution

Initially, $Q_R = 4\pi R^2\sigma$ & $Q_{2R} = 4\pi(2R)^2\sigma = 16\pi R^2\sigma$

When the spheres are connected by wire,

$$V'_R = \frac{1}{4\pi \in_0}\frac{Q'_R}{R} \text{ \& } V'_{2R} = \frac{1}{4\pi \in_0}\frac{Q'_{2R}}{2R}$$

Since potentials are equal now, $\frac{Q'_R}{Q'_{2R}} = \frac{1}{2}$

$$\therefore \frac{Q'_R + Q'_{2R}}{Q'_{2R}} = \frac{1+2}{2} = \frac{3}{2}$$

Since charge is conserved, $Q'_R + Q'_{2R} = Q_R + Q_{2R} = 20\pi R^2\sigma$

$$\therefore \frac{Q'_R + Q'_{2R}}{Q'_{2R}} = \frac{20\pi R^2\sigma}{4\pi(2R)^2\sigma'_{2R}} = \frac{3}{2}$$

$$\therefore \frac{5\sigma}{4\sigma'_{2R}} = \frac{3}{2}$$

$$\therefore \sigma'_{2R} = \frac{5}{6}\sigma$$

Hence, (A)