$L = \lim_{n\to\infty} \frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{n}$. Then,

(A) $< \frac{\pi}{4}$ (B) $> \frac{\pi}{4} + \frac{1}{2}$

(C) $< \frac{\pi}{4} + \frac{1}{2}$ (D) $> \frac{\pi}{4}$

Select correct option(s).

Solution

$$L = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{n+r}{n^2+r^2}$$

$$\therefore L = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1+\frac{r}{n}}{1+\left(\frac{r}{n}\right)^2} \cdot \frac{1}{n}$$

$$\therefore L = \int_0^1 \frac{1+x}{1+x^2}\,dx = \int_0^1 \frac{1}{1+x^2}\,dx + \frac{1}{2}\int_0^1 \frac{2x}{1+x^2}\,dx$$

$$\therefore L = \tan^{-1} x\Big|_0^1 + \frac{1}{2}\ln(1+x^2)\Big|_0^1 = \frac{\pi}{4} + \frac{\ln 2}{2}$$

$$\therefore L = \frac{\pi}{4} + \frac{0.693}{2}$$

Hence, (C) & (D)