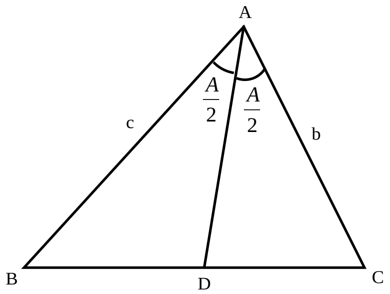
In a triangle ABC, the length of the bisector of angle A is

(i) $\dfrac{2bc\sin\frac{A}{2}}{b+c}$ (ii) $\dfrac{2bc\cos\frac{A}{2}}{b+c}$ (iii) $\dfrac{2bc\,\mathrm{cosec}\frac{A}{2}}{b+c}$ (iv) $\dfrac{2bc\sec\frac{A}{2}}{b+c}$



Solution

Sine rule in triangle ADB yields, $\dfrac{AD}{\sin B}=\dfrac{BD}{\sin\frac{A}{2}}$

Since AD bisects angle A, $\dfrac{BD}{c}=\dfrac{CD}{b}$

Now, $\dfrac{BD}{c}=\dfrac{CD}{b}=\dfrac{BD+CD}{c+b}=\dfrac{a}{b+c}$

$\therefore BD=\dfrac{ac}{b+c}$

Now, $\dfrac{AD}{\sin B}=\dfrac{BD}{\sin\frac{A}{2}}=\dfrac{ac}{b+c}.\dfrac{1}{\sin\frac{A}{2}}$

$\therefore AD=\dfrac{ac}{b+c}.\dfrac{\sin B}{\sin\frac{A}{2}}=\dfrac{ac}{b+c}.\dfrac{b\sin A}{a\sin\frac{A}{2}}$ $\left[\because \dfrac{\sin B}{b}=\dfrac{\sin A}{a}\right]$

$\therefore AD=\dfrac{ac}{b+c}.\dfrac{b.2\sin\frac{A}{2}\cos\frac{A}{2}}{a\sin\frac{A}{2}}=\dfrac{2bc\cos\frac{A}{2}}{b+c}$

Hence, (ii)