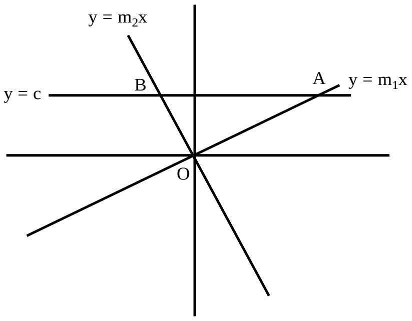
If m_1 and m_2 are the roots of the equation

$$x^2 + (\sqrt{3}+2)x + (\sqrt{3}-1) = 0$$

the area of the triangle formed by the lines $y = m_1x$, $y = m_2x$ and $y = c$ is ($k = \sqrt{11}$),

(A) $\frac{\sqrt{3}-1}{4}kc^2$ (B) $\frac{\sqrt{3}+1}{4}kc^2$ (C) $\frac{\sqrt{3}-1}{2}kc^2$ (D) $\frac{\sqrt{3}+1}{2}kc^2$

Solution



$A \equiv \left(\frac{c}{m_1}, c\right)$, $B \equiv \left(\frac{c}{m_2}, c\right)$

Area of triangle OAB = $\frac{1}{2} \times \left|\frac{c}{m_1} - \frac{c}{m_2}\right| \times |c|$ = $\frac{1}{2}\left|\frac{m_2 - m_1}{m_1 m_2}\right| c^2$

Since m_1 and m_2 are the roots of the equation $x^2 + (\sqrt{3}+2)x + (\sqrt{3}-1) = 0$,

$m_1 + m_2 = -(\sqrt{3}+2)$ & $m_1 m_2 = \sqrt{3}-1$

Now, $(m_2 - m_1)^2 = (m_2 + m_1)^2 - 4m_2m_1 = (\sqrt{3}+2)^2 - 4(\sqrt{3}-1)$

$\therefore (m_2 - m_1)^2 = 3 + 4\sqrt{3} + 4 - 4\sqrt{3} + 4 = 11 = k^2$

$\therefore m_2 - m_1 = \pm k$

Now, area of triangle OAB = $\frac{1}{2}\left|\frac{m_2 - m_1}{m_1 m_2}\right| c^2$ = $\frac{1}{2}\left|\frac{\pm k}{\sqrt{3}-1}\right| c^2$ = $\frac{\sqrt{3}+1}{4}kc^2$

Hence, (B)