If a, b and c are all different from zero, and

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

is equal to zero, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is

(A) abc (B) $\frac{1}{abc}$ (C) 1 (D) -1

Solution

$$\Delta = \begin{vmatrix} a & 1 & 1 \\ -b & 1+b & 1 \\ 0 & 1 & 1+c \end{vmatrix}_{C_1 \to C_1 - C_2}$$

Further, $\Delta = \begin{vmatrix} a & 0 & 1 \\ -b & b & 1 \\ 0 & -c & 1+c \end{vmatrix}_{C_2 \to C_2 - C_3}$

$\therefore \Delta = a\{b(1+c)+c\}+bc = 0$ [expanding about R_1]

$\therefore a(b+bc+c)+bc = ab+bc+ca+abc = 0$

$$\therefore abc\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}+1\right)=0$$

Since a, b, c all are different from zero, $\frac{1}{c}+\frac{1}{a}+\frac{1}{b}+1=0$

$$\therefore \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=-1$$

Hence, (D)