If R_E be the radius of Earth, then the ratio of the acceleration due to gravity at a depth 'r' below and a height 'r' above the Earth surface is:

(A) $1+\frac{r}{R_E}+\frac{r^2}{{R_E}^2}+\frac{r^3}{{R_E}^3}$ (B) $1-\frac{r}{R_E}-\frac{r^2}{{R_E}^2}-\frac{r^3}{{R_E}^3}$

(C) $1+\frac{r}{R_E}-\frac{r^2}{{R_E}^2}-\frac{r^3}{{R_E}^3}$ (D) $1+\frac{r}{R_E}-\frac{r^2}{{R_E}^2}+\frac{r^3}{{R_E}^3}$

Solution

$g'=\frac{GM}{{R_E}^3}(R_E-r)$ for 'r' below the surface

$g''=\frac{GM}{(R_E+r)^2}$ for 'r' above the surface

$$\frac{g'}{g''}=\frac{(R_E-r)(R_E+r)^2}{{R_E}^3}=\frac{({R_E}^2-r^2)(R_E+r)}{{R_E}^3}$$

$$\therefore \frac{g'}{g''}=\frac{{R_E}^3+{R_E}^2r-r^2R_E-r^3}{{R_E}^3}=1+\frac{r}{R_E}-\left(\frac{r}{R_E}\right)^2-\left(\frac{r}{R_E}\right)^3$$

Hence, (C)