

Suppose $f'(x)$ exists for each x and $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for every real number x . Then,

- (A) h is increasing whenever f is increasing
- (B) h is increasing whenever f is decreasing
- (C) h is decreasing whenever f is decreasing
- (D) h is decreasing whenever f is increasing

Select correct option(s).

Solution

$$h'(x) = f'(x) - 2f(x)f'(x) + 3\{f(x)\}^2 f'(x)$$

$$\therefore h'(x) = f'(x)[3\{f(x)\}^2 - 2f(x) + 1]$$

$$\therefore h'(x) = 3f'(x) \left[\{f(x)\}^2 - 2f(x) \cdot \frac{1}{3} + \frac{1}{3} \right]$$

$$\therefore h'(x) = 3f'(x) \left[\{f(x)\}^2 - 2f(x) \cdot \frac{1}{3} + \frac{1}{9} + \frac{2}{9} \right]$$

$$\therefore h'(x) = 3f'(x) \left[\left\{ f(x) - \frac{1}{3} \right\}^2 + \frac{2}{9} \right]$$

$$\therefore h'(x) = f'(x)(+ve)$$

$$\therefore h'(x) > 0 \text{ when } f'(x) > 0 \text{ \& } h'(x) < 0 \text{ when } f'(x) < 0$$

Hence, (A) & (C)