For an ideal gas, the instantaneous change in pressure 'p' with volume 'V' is given by the equation $\frac{dp}{dV} = -ap$ (a > 0). If p = p_0 at V = 0 is the given boundary condition, then the maximum temperature one mole of gas can attain is: (R = gas constant)

(A) $0^\circ C$ (B) $\frac{ap_0}{eR}$ (C) $\frac{p_0}{aeR}$ (D) ∞

Solution

We have, $\frac{dp}{dV} = -ap$

$$\therefore \int \frac{dp}{p} = \int -adV$$

$$\therefore \ln p = -aV + C$$

When V = 0, p = p_0 $\therefore \ln p_0 = 0 + C$

Thus, $\ln p = -aV + \ln p_0$

$$\therefore \ln \frac{p}{p_0} = -aV$$

For 1 mole ideal gas, $T = \frac{pV}{R} = -\frac{p \ln \frac{p}{p_0}}{aR}$

Let $f(p) = p \ln \frac{p}{p_0}$, so when T is maximum f(p) is minimum due to -ve sign.

$f'(p) = p \frac{1/p_0}{p/p_0} + \ln \frac{p}{p_0} = 1 + \ln \frac{p}{p_0} = 0$ for f(p) to be minima/maxima

$$\therefore p = \frac{p_0}{e}$$

$$f''(p) = \frac{1/p_0}{p/p_0} = \frac{1}{p} > 0$$

From the second derivative test it can be seen that $p = \frac{p_0}{e}$ corresponds to minima for f(p) and thus maxima for T.

$$T_{\max} = -\frac{(p_0/e)\ln \frac{(p_0/e)}{p_0}}{aR} = \frac{p_0}{aeR}$$

Hence, (C)

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