Equation of circle passing through (1, 0) and (0, 1) having smallest possible radius is given by,

(A) $x^2+y^2+x+y=0$ (B) $x^2+y^2+x-y=0$

(C) $x^2+y^2-x+y=0$ (D) $x^2+y^2-x-y=0$

Solution

Let equation of the circle be $x^2+y^2+2gx+2fy+c=0$

Since it passes through (1, 0), 1+2g+c=0

Since it also passes through (0, 1), 1+2f+c=0

From the above two equations, g = f & c = -1-2g

Now, $r^2=g^2+f^2-c=g^2+g^2-(-1-2g)=2g^2+2g+1$

$$\therefore r^2=2\left(g^2+2.\frac{1}{2}g+\frac{1}{2}\right)=2\left(g^2+2.\frac{1}{2}g+\frac{1}{4}+\frac{1}{4}\right)$$

$$\therefore r^2=2\left\{\left(g+\frac{1}{2}\right)^2+\frac{1}{4}\right\}$$

So, radius is smallest when $g=-\frac{1}{2}$

$\therefore f=-\frac{1}{2}$ & c = -1-2g = 0

Equation of circle is $x^2+y^2+(-1)x+(-1)y+0=0$

Hence, (D)