If a, b and c are three positive real numbers, then the minimum value of the expression

$$\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}$$

is

(A) 1 (B) 2 (C) 3 (D) None of these

Solution

The given expression can be written as $\left(\frac{b}{a}+\frac{a}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)+\left(\frac{c}{b}+\frac{b}{c}\right)$

We have, $AM \geq GM$ for positive numbers.

So, $\frac{\frac{b}{a}+\frac{a}{b}}{2} \geq \sqrt{\frac{a}{b}.\frac{b}{a}}$

$\Rightarrow \frac{b}{a}+\frac{a}{b} \geq 2$

Similarly, $\frac{c}{a}+\frac{a}{c} \geq 2$ & $\frac{c}{b}+\frac{b}{c} \geq 2$

Adding we get, $\left(\frac{b}{a}+\frac{a}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)+\left(\frac{c}{b}+\frac{b}{c}\right) \geq 2+2+2$

So, $\left(\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}\right)_{\min} = 6$

Hence, (D)