Area of triangle inscribed in the parabola $y^2 = 4ax$ (a > 0) having y_1, y_2 and y_3 as the ordinates of vertices is:

(A) $\frac{1}{2}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

(B) $\frac{1}{2a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

(C) $\frac{1}{4a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

(D) $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

Solution

Let vertices of the triangle be $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ & $(at_3^2, 2at_3)$

Area of triangle Δ = absolute value of $\frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix} = \frac{a.2a}{2}\begin{vmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{vmatrix}_{Abs}$

$$\therefore \Delta = a^2 \begin{vmatrix} t_1^2 - t_3^2 & t_1 - t_3 & 0 \\ t_2^2 - t_3^2 & t_2 - t_3 & 0 \\ t_3^2 & t_3 & 1 \end{vmatrix}_{Abs(R_1 \to R_1 - R_3, R_2 \to R_2 - R_3)}$$

$$\therefore \Delta = a^2\{(t_1 - t_3)(t_2 - t_3)\}_{Abs} \begin{vmatrix} t_1 + t_3 & 1 & 0 \\ t_2 + t_3 & 1 & 0 \\ t_3^2 & t_3 & 1 \end{vmatrix}_{Abs}$$

$$\therefore \Delta = a^2\{(t_1 - t_3)(t_2 - t_3)\}_{Abs} \begin{vmatrix} t_1 + t_3 & 1 \\ t_2 + t_3 & 1 \end{vmatrix}_{Abs}$$

$$\therefore \Delta = a^2 \,|(t_1 - t_3)(t_2 - t_3)(t_1 - t_2)|$$

$$\therefore \Delta = a^2 \left|\left(\frac{y_1}{2a} - \frac{y_3}{2a}\right)\left(\frac{y_2}{2a} - \frac{y_3}{2a}\right)\left(\frac{y_1}{2a} - \frac{y_2}{2a}\right)\right|$$

$$\therefore \Delta = \frac{1}{8a}\left|(y_1 - y_3)(y_2 - y_3)(y_1 - y_2)\right|$$

Hence, (D)