A leaky parallel plate capacitor is filled completely with a material having dielectric constant k = 5 and electric conductivity $\sigma = 7.4 \times 10^{-12} \Omega^{-1} m^{-1}$. If the charge on the plate at the instant t = 0 is $q = 8.85 \mu C$, then the leakage current at the instant t = 12 s is approximately given by:

(A) $1.5\mu A$ (B) $0.2\mu A$ (C) $1\mu A$ (D) 0

Solution

We have, $q = q_0 e^{-t/\tau}$

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau}$$
Now, $\tau = RC = \frac{\rho l}{A} \cdot \frac{k \in_0 A}{d} = \frac{d}{\sigma A} \cdot \frac{k \in_0 A}{d} = \frac{k \in_0}{\sigma}$

$$\frac{q_0}{\tau} = \frac{q_0 \sigma}{k \in_0} = \frac{8.85 \times 10^{-6} \times 7.4 \times 10^{-12}}{5 \times 8.85 \times 10^{-12}} = \frac{7.4}{5} \times 10^{-6}$$

$$\frac{t}{\tau} \Big|_{t=12s} = \frac{12\sigma}{k \in_0} = \frac{12 \times 7.4 \times 10^{-12}}{5 \times 8.85 \times 10^{-12}} = \frac{12 \times 7.4}{5 \times 8.85} \approx 2$$
So, $i = \frac{q_0}{\tau} e^{-t/\tau} \approx \frac{7.4}{5} \times 10^{-6} \times e^{-2} = \frac{7.4}{5e^2} \mu A \approx \frac{7.4}{5 \times 7.4} \mu A = 0.2 \mu A$

Option (B)