$$8\sin^3\frac{\pi}{14}-4\sin^2\frac{\pi}{14}-4\sin\frac{\pi}{14}=$$

(A) 0 (B) -1 (C) 1 (D) $-\frac{1}{4}$

Solution

The given expression = $8\sin^3\frac{\pi}{14}-4\sin\frac{\pi}{14}-2\times 2\sin^2\frac{\pi}{14}$

$$=4\sin\frac{\pi}{14}\left(2\sin^2\frac{\pi}{14}-1\right)-2\times\left(1-\cos\frac{\pi}{7}\right)$$

$$=-4\sin\frac{\pi}{14}\cos\frac{2\pi}{14}+2\cos\frac{\pi}{7}-2$$

$$=-2\left(\sin\frac{3\pi}{14}-\sin\frac{\pi}{14}\right)+2\cos\frac{\pi}{7}-2$$

$$=2\sin\frac{\pi}{14}-2\sin\frac{3\pi}{14}+2\cos\frac{\pi}{7}-2$$

$$=2\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)-2\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)+2\cos\frac{\pi}{7}-2$$

$$=2\cos\frac{3\pi}{7}-2\cos\frac{2\pi}{7}+2\cos\frac{\pi}{7}-2$$

$$=2\cos\frac{\pi}{7}+2\cos\frac{3\pi}{7}+2\cos\frac{5\pi}{7}-2$$

$$=\frac{1}{\sin\frac{\pi}{7}}\left(2\sin\frac{\pi}{7}\cos\frac{\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{3\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{5\pi}{7}\right)-2$$

$$=\frac{1}{\sin\frac{\pi}{7}}\left(\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}-\sin\frac{2\pi}{7}+\sin\frac{6\pi}{7}-\sin\frac{4\pi}{7}\right)-2$$

$$=\frac{1}{\sin\frac{\pi}{7}}\left(\sin\frac{6\pi}{7}\right)-2=1-2=-1$$

Hence, (B)