The minimum value of $64\sec\theta + 27\operatorname{cosec}\theta$ when θ lies in $\left(0, \frac{\pi}{2}\right)$ is,

(A) at $\theta = 63^{\circ}$ approximately
(B) at $\theta = 37^{\circ}$ approximately
(C) at $\theta = 45^{\circ}$
(D) equal to 125

Select correct option(s).

Solution

Let $f(\theta) = 64\sec\theta + 27\operatorname{cosec}\theta$

$f'(\theta) = 64\sec\theta\tan\theta - 27\operatorname{cosec}\theta\cot\theta = 0$ for max./min.

$\therefore 64\sin^3\theta = 27\cos^3\theta$

$\Rightarrow \tan\theta = \frac{3}{4}$

$f''(\theta) = 64\sec\theta\tan^2\theta + 64\sec^3\theta + 27\operatorname{cosec}\theta\cot^2\theta + 27\operatorname{cosec}^3\theta$

$\because \theta \in \left(0, \frac{\pi}{2}\right), f''(\theta) > 0$

So, there is minima when $\tan\theta = \frac{3}{4}$ or $\sin\theta = \frac{3}{5}$ & $\cos\theta = \frac{4}{5}$

$f_{\min}(\theta) = 64 \times \frac{5}{4} + 27 \times \frac{5}{3} = 125$

Hence, (B) & (D)

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