A proton (mass m) is headed with speed u towards another stationary proton so as to have head-on collision if possible. However, due to sufficient initial distance between them the collision is avoided. The shortest distance attained between the two protons is: [e = charge on proton]

(A) $\frac{e^2}{\pi \in_0 mu^2}$ (B) $\frac{e}{\pi \in_0 mu^2}$ (C) $\frac{e}{\pi \in_0 mu}$ (D) $\frac{e^2}{\pi \in_0 mu}$

Solution

As the moving proton heads towards the initially stationary proton, the electrostatic repulsion makes the initially stationary proton move. The fast proton slows down and the slow proton speeds up. At shortest distance d, both protons have same instantaneous speed (say v).

Conservation of linear momentum gives,

$$mu = mv + mv$$

$$\therefore v = \frac{u}{2}$$

Conservation of energy gives,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi \in_0}\frac{e^2}{d}$$

$$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{4}mu^2 + \frac{1}{4\pi \in_0}\frac{e^2}{d}$$

$$\Rightarrow d = \frac{1}{m\pi \in_0}\frac{e^2}{u^2}$$

Hence, (A)