A flywheel with an initial angular velocity $\omega_0$, decelerates due to forces whose moment, relative to the axis is proportional to the square root of angular velocity. The average angular velocity over the total time of deceleration after which the flywheel comes to rest is given by,

(A) $\frac{\sqrt{\omega_0}}{3}$ (B) $\frac{\omega_0}{3}$ (C) $\frac{2\omega_0}{3}$ (D) $\frac{3\omega_0}{2}$

*Solution*

We have, angular acceleration $\alpha \propto -\sqrt{\omega}$

Or, $\alpha = \frac{d\omega}{dt} = -k\sqrt{\omega}$

$\therefore \frac{d\omega}{\sqrt{\omega}} = -kdt$ ……….(*)

Now, $\omega_{av} = \frac{\int_0^T \omega dt}{T}$

From (*), $\frac{\omega}{-k}\frac{d\omega}{\sqrt{\omega}} = \omega dt$

So, $\omega_{av} = \frac{\int_0^T \frac{\omega}{-k}\frac{d\omega}{\sqrt{\omega}}}{T} = \frac{\int_{\omega_0}^0 \sqrt{\omega}d\omega}{-kT}$

$$\omega_{av} = \frac{\omega^{3/2}\Big|_{\omega_0}^{0}}{-kT(3/2)} = \frac{2\omega_0^{3/2}}{3kT}$$

Let's use (*) again to find kT

We have, $\int_{\omega_0}^0 \frac{d\omega}{\sqrt{\omega}} = \int_0^T -kdt$

$\therefore 2\sqrt{\omega}\Big|_{\omega_0}^0 = -kT$

$\Rightarrow -2\sqrt{\omega_0} = -kT$ Or $kT = 2\sqrt{\omega_0}$

Now, $\omega_{av} = \frac{2\omega_0^{3/2}}{3.2\sqrt{\omega_0}} = \frac{\omega_0}{3}$

Hence, (B)