Select correct option(s):

(A) $222^{333} > 333^{222}$ (B) $222^{333} < 333^{222}$
(C) $222^{333} = 333^{222}$ (D) $222^{333} \neq 333^{222}$

Solution

We have to compare $222^{333} \,\&\, 333^{222}$

Or, we have to compare $2\times111^{3\times111} \,\&\, 3\times111^{2\times111}$

Let 111 = a

So, we have to compare $(2a)^{3a} \,\&\, (3a)^{2a}$

Comparing with log to the base e is justified as ln is an increasing function.

So, we have to compare $3a\ln(2a) \,\&\, 2a\ln(3a)$

Or, we have to compare $\dfrac{\ln(2a)}{2a} \,\&\, \dfrac{\ln(3a)}{3a}$

Let, $\dfrac{\ln x}{x} = f(x)$

$$f'(x) = \frac{x.\frac{1}{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$$

f'(x) < 0 as x is quite large for which ln x > 1

Thus, f(x) is a decreasing function.

So, f(2a) > f(3a)

$$\therefore \frac{\ln(2a)}{2a} > \frac{\ln(3a)}{3a}$$

$$\Rightarrow 3a\ln(2a) > 2a\ln(3a)$$

$$\Rightarrow \ln(2a)^{3a} > \ln(3a)^{2a} \quad \text{Or} \quad (2a)^{3a} > (3a)^{2a}$$

$$\therefore 222^{333} > 333^{222}$$

Hence, (A) & (D)