Evaluate,

$$I = \int \frac{\sec^2 x}{(\sec x + \tan x)^5} dx$$

Solution

$$I = \int \frac{\cos^3 x}{(1+\sin x)^5} dx$$

Let, sin x = t

So, $I = \int \frac{1-t^2}{(1+t)^5} dt = \int \frac{1-t}{(1+t)^4} dt$

Further, $I = -\int \frac{t-1}{(1+t)^4} dt = -\int \frac{t+1-2}{(1+t)^4} dt$

$$\therefore I = -\int \frac{1}{(1+t)^3} - \frac{2}{(1+t)^4} dt = 2\int \frac{dt}{(1+t)^4} - \int \frac{dt}{(1+t)^3}$$

$$\therefore I = 2\frac{(1+t)^{-4+1}}{-4+1} - \frac{(1+t)^{-3+1}}{-3+1} = \frac{1}{2(1+t)^2} - \frac{2}{3(1+t)^3}$$

$$\therefore I = \frac{1}{2(1+\sin x)^2} - \frac{2}{3(1+\sin x)^3} + C$$