If a, b > 0 and

$$\Delta = \begin{vmatrix} x & a & a \\ b & x & a \\ b & b & x \end{vmatrix}$$

then which of the following option(s) is/are correct?

(A) Δ has a local maximum at $x = \sqrt{ab}$

(B) Δ has a local minimum at $x = \sqrt{ab}$

(C) Δ has a local maximum at $x = -\sqrt{ab}$

(D) Δ has a local minimum at $x = -\sqrt{ab}$

Solution

$$\Delta' = \begin{vmatrix} 1 & 0 & 0 \\ b & x & a \\ b & b & x \end{vmatrix} + \begin{vmatrix} x & a & a \\ 0 & 1 & 0 \\ b & b & x \end{vmatrix} + \begin{vmatrix} x & a & a \\ b & x & a \\ 0 & 0 & 1 \end{vmatrix}$$

$$\therefore \Delta' = (x^2 - ab) + (x^2 - ab) + (x^2 - ab) = 3(x^2 - ab)$$

Equating $\Delta' = 0$ yields $x = \pm\sqrt{ab}$

Further, $\Delta'' = 6x$

So $\Delta'' > 0$ for $x = \sqrt{ab}$ which means local minimum

& $\Delta'' < 0$ for $x = -\sqrt{ab}$ which means local maximum

Hence, (B) & (C)