

For a triangle ABC having area Δ , the expression $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2} =$

- (1) a^2 (2) $\sin^2 A$ (3) $\sin A$ (4) $2\left(\frac{\Delta}{bc}\right)^2$

Solution

$$\text{The given expression} = \frac{2s(2s-a-a)(2s-b-b)(2s-c-c)}{4b^2c^2}$$

$$= \frac{16s(s-a)(s-b)(s-c)}{4b^2c^2}$$

$$= 4 \frac{\Delta^2}{b^2c^2} = 4 \times \frac{1}{4} \sin^2 A \left[\because \Delta = \frac{1}{2}bc \sin A, \frac{\Delta}{bc} = \frac{1}{2} \sin A \right]$$

$$= \sin^2 A$$

Hence, (2)