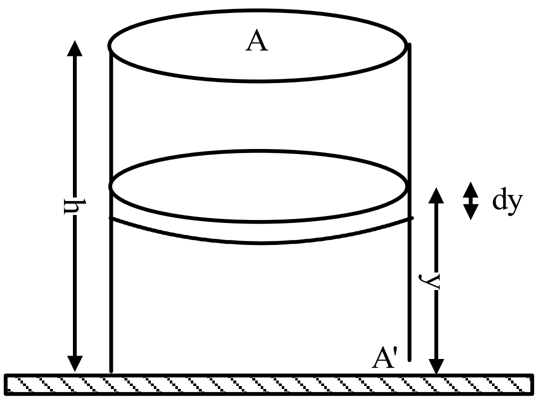
A cylindrical vessel of cross-sectional area A has water filled upto height h. There is a small hole at the bottom having cross-sectional area A'. The time taken to empty the vessel is given by,

(A) $\frac{A}{A'}\sqrt{\frac{2h}{g}}$ (B) $\frac{2A}{A'}\sqrt{\frac{h}{g}}$ (C) $\frac{A}{A'}\sqrt{\frac{h}{2g}}$ (D) $\frac{A'}{A}\sqrt{\frac{2h}{g}}$

Solution



Let's say at any time t the height of water is y and the speed is v. In time dt the height changes by dy and the volume change is dV.

Then, volume of water coming out per second at instant t $=\left|\frac{dV}{dt}\right| = A'v$

Or, $\frac{dV}{dt} = -A'v = -A'\sqrt{2gy}$

Also, $dV = Ady$ (note that dy is -ve and so is dV)

$$\therefore A\frac{dy}{dt} = -A'\sqrt{2gy}$$

$$\therefore \frac{A'}{A}dt = -\frac{dy}{\sqrt{2gy}}$$

After integration we have,

$$\frac{A'}{A}t\Big|_0^T = -\frac{1}{\sqrt{2g}}2\sqrt{y}\Big|_h^0$$

$$\therefore \frac{A'}{A}T = -\frac{1}{\sqrt{2g}}(-2\sqrt{h}) = \sqrt{\frac{2h}{g}}$$

$$\therefore T = \frac{A}{A'}\sqrt{\frac{2h}{g}}$$

Hence, (A)