Let

$$f(x)=\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$$

The value which should be assigned to f at x = 0 so that it is continuous everywhere is,

(A) -1 (B) $\frac{1}{2}$ (C) 1 (D) 2

Solution

Since the function is continuous everywhere, it should be continuous at x = 0 also.

So, $f(0)=\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

Or, $f(0)=\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}\times\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$

Or, $f(0)=\lim_{x\to 0}\frac{(1+\sin x)-(1-\sin x)}{x}\times\frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$

Or, $f(0)=2\lim_{x\to 0}\frac{\sin x}{x}\times\frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$

Or, $f(0)=2\lim_{x\to 0}\frac{\sin x}{x}\times\lim_{x\to 0}\frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$

$\therefore\ f(0)=2\times 1\times\frac{1}{2}=1$

Hence, (C).