The range of the function

$$f(x)=\frac{x^2}{x^4+1}$$ is

(A) $\left(0,\frac{1}{2}\right)$ (B) $\left[0,\frac{1}{2}\right]$ (C) $\left[\frac{1}{2},2\right]$ (D) $[0,2]$

Solution

Considering even powers on x, it is clear that $f(x)\geq 0$.

Further, $x^4+1=(x^2-1)^2+2x^2$

So, $x^4+1\geq 2x^2$

Or, $\frac{x^2}{x^4+1}\leq\frac{1}{2}$

$\therefore 0\leq f(x)\leq\frac{1}{2}$

Hence, (B)