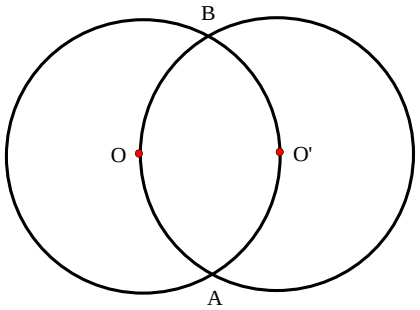
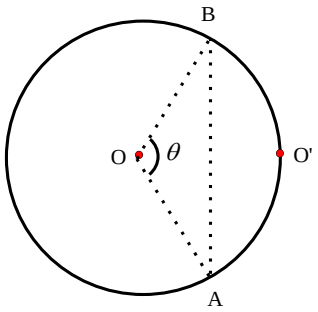
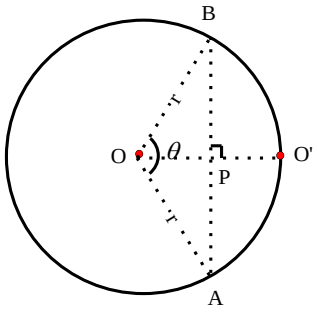
Find the area of the region OAO'B bounded between the identical circles each having radius r.



Solution



Area of segment AO'B = $\frac{\theta}{360}\times\pi r^2 -$ Area of triangle OAB



OO' = r = 2OP

$\cos\frac{\theta}{2}=\frac{OP}{OB}=\frac{r/2}{r}=\frac{1}{2}$, $\theta=120^\circ$

Area of segment AO'B = $\frac{120}{360}\times\pi r^2-\frac{1}{2}r\times r\times\sin 120^\circ=\frac{1}{3}\pi r^2-\frac{\sqrt{3}}{4}r^2$

Required area = 2 x Area of segment AO'B = $2\times\left(\frac{1}{3}\pi r^2-\frac{\sqrt{3}}{4}r^2\right)=2r^2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$