# NEWTON'S LAWS OF MOTION, FRICTION

**Contents**

Newton's laws of motion, Types of forces, Pseudo forces, Pulleys, Lift problem, Motion of connected particles, Inclined planes, Friction etc.

**Newton's Laws of Motion**

Change of inertia (rest or motion) and direction (in case of motion) is only possible by an external force in an inertial frame of reference. Inertial frame of reference is the frame which is a non-accelerated frame. Earth is treated as inertial frame with some degree of approximation.

$\vec{F}_{net} = m.\vec{a} = m\dfrac{d\vec{v}}{dt}$ if mass is constant.  In general,  $\vec{F}_{net} = \dfrac{d(m\vec{v})}{dt}$. This is valid for an inertial frame of reference.

For every action force there is an equal and opposite reaction force where these forces act on different bodies. If one body is considered as the system out of two bodies then these forces don't cancel each other as the other force is acting on different body. However, in some problems system as a whole approach is applied, i.e. bodies experiencing action and reaction forces are together considered as part of a system and in such cases such action-reaction forces become internal forces and cancel out. Several objects or particles can be treated as a system if the relative distance between them does not change and if their direction of motion and acceleration is same.

**Types of Forces**

Gravitational force acts between any two masses kept anywhere in the universe.  It follows inverse square rule ( $F \propto \dfrac{1}{\text{distance}^2}$ ) and is attractive in nature.

$F = \dfrac{GM_1M_2}{R^2}$ .

The force mg, which Earth applies on the bodies, is gravitational force.

Electroweak force acts between any two charges and can be attractive or repulsive in nature.

Electrostatic force is one kind of electroweak force and is given by $F = K \frac{q_1 q_2}{r^2}$. As can be seen it also follows inverse square law.

Gravitational force is normally considered to be analogous to electrostatic Coulomb's force. In case of Gravitational force, G is universal constant. In case of Coulomb's force, the constant K is not universal. K depends on the medium. When you hit the table, you feel pain; this is due to the normal reaction force, which is actually electroweak force. When you rub your hands in winter, you feel warmth, which is due to the work done by the frictional force. Frictional force is also an electroweak force. When you play tug-of-war then the force which rope applies on your opponent is an electroweak force. The force of buoyancy responsible to cause floating is an electroweak force. Most of the forces that you experience are electroweak forces.

Nuclear force is a short-range attractive force (very strong), which is responsible for the existence of the nucleus as without it protons inside the nucleus would fly apart due to repulsion. It does not follow inverse square rule and the mathematical expression for nuclear force is still now known for sure.

**Pseudo Forces**

A person sitting in an accelerated frame (non-inertial frame) experiences pseudo forces. Eg. when you take sharp turn while riding on bike, you feel an outward force which is called centrifugal force - a typical example of pseudo force. Pseudo forces exist only in non-inertial (accelerated) frames. Pseudo forces should not be considered for inertial frame.

If acceleration of the non-inertial frame is $\vec{a}$ and the mass of object under consideration is m, then the pseudo force on m is $-m\vec{a}$.

**String**

When a block is hanging from string, it pulls the string down. The string also applies force on the block upwards. The force applied by the string is called tension. If the string is light and isolated then the tension is same throughout it. However, when the string is heavy – like rope – then the tension is non uniform. Generally, we consider the string to be unstretchable. Tension is an electromagnetic force.

## Spring

When spring is under elongation or compression x, it applies force which is proportional to x. This proportionality sign is replaced by constant k which is called spring constant. Spring constant depends on the spring. Spring force is an electroweak force whose magnitude is given by kx. The direction of spring force is opposite to $\vec{x}$ and due to this reason; it is a common practice to write $\vec{F} = -k\vec{x}$ .

## Pulley

Pulley has an axle around which it rotates and a string passes over the pulley. The axle is connected to the support. The string is generally connected to external objects. If pulley is light and frictionless then tension in the string on both sides is same. When it is said that the pulley is frictionless it implies that there is no friction with the axle and on its surface (it may be amusing to visualise that the string is slipping as when there is no friction on the surface this is what would happen, this is what is the case in most problems).

Note: In case an object is light then there can be no net force acting on it, but it can accelerate without any force.

## Pulleys Case-1

Writing equation of motion for $m_1$,
$T - m_1 g = m_1 a$ ........(1)

Writing equation of motion for $m_2$,
$m_2 g - T = m_2 a$ ........(2)

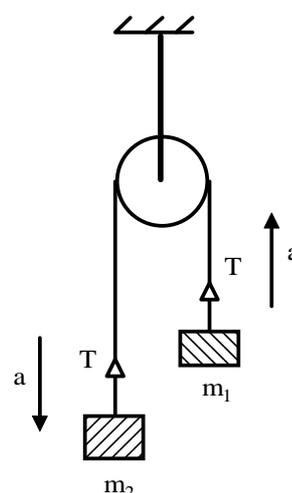Solving the above two equations,

Acceleration of masses, $a = \left( \dfrac{m_2 - m_1}{m_2 + m_1} \right) g$ .

Acceleration of the centre of mass $a_{CM} = \left( \dfrac{m_2 - m_1}{m_2 + m_1} \right)^2 g$ .

Velocity of masses can be obtained by the following relation obtained from mechanical energy conservation:
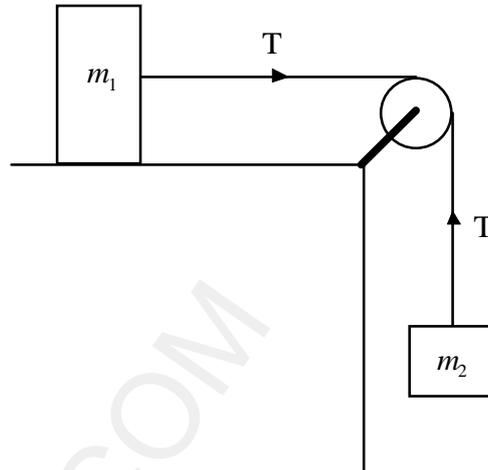
$(m_2 - m_1) gh = \dfrac{1}{2} (m_1 + m_2) v^2$ .

Atwood Machine

## Pulleys Case-2

Using, $m_2g - T = m_2a$ and

$T = m_1a$.

Solving for $a = \dfrac{m_2 g}{m_1 + m_2}$.

## Pulleys Case-3 (Movable Pulley)

Let acceleration of pulley be **a'** w.r.t. ground.

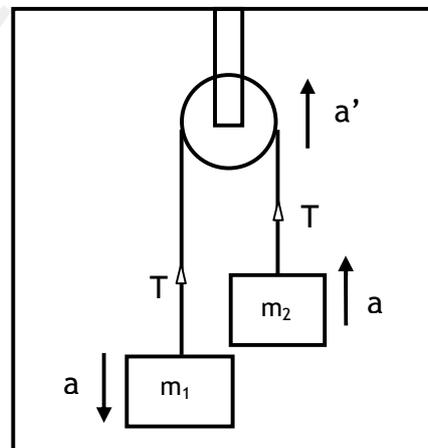Let **a** be the acceleration of the blocks w.r.t. the pulley

Consider the observer at the pulley (i.e. non-inertial frame). Now, pseudo forces $m_1a'$ and $m_2a'$ would need to be considered in downward direction.
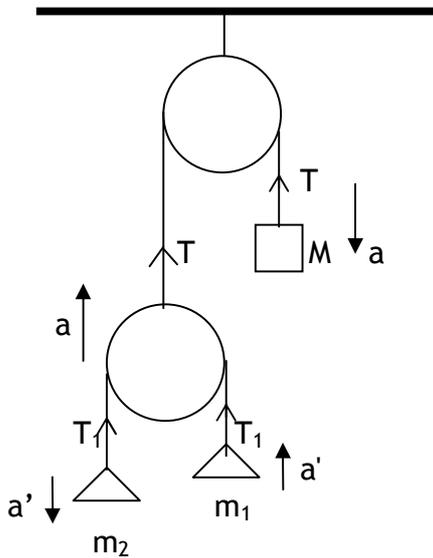
$m_1(g+a') - T = m_1a$,

$T - m_2(g+a') = m_2a$,

solving for $a' = \dfrac{(m_2 - m_1)}{m_2 + m_1}(g + a)$

and $T = \dfrac{2m_1m_2}{m_1 + m_2}(g + a)$.

## Pulleys Case-4 (Multiple Pulleys)

We can use the following equations.

$Mg - T = Ma$ -------- (1)

"a" is the acceleration of M w.r.t. ground

$T = 2T_1$ --------(2)

Let a' be the acceleration of $m_1$ and $m_2$ w.r.t. movable pulley. Consider the observer at movable pulley which is a non-inertial system,

$m_2(g+a) - T_1 = m_2a'$ ------(3)

$T_1 - m_1(g+a) = m_1a'$

Solve for $T_1$, T and a, a'.

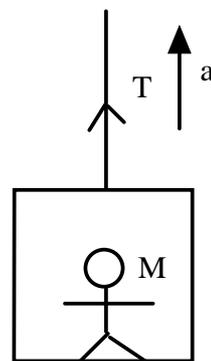## Lift Problem

Mg acts downwards

Normal reaction from the floor acts upwards

N−Mg=Ma

$\Rightarrow N = M(a+g)$

Apparent mass,

$M' = M\dfrac{a+g}{g}$

Elevator can accelerate downwards also. In general, we have

Apparent mass of the man $M' = M\dfrac{g\pm a}{g}$

The above result has to do with acceleration; it has nothing to do with direction of velocity.

Tension in the cable $T = M_T(g \pm a)$ where $M_T$ is the mass of the cabin + mass of the man combined.

With lift not accelerating, $a = 0$ so apparent mass equals to actual mass and $T = Mg$. Also when the lift moves with a constant velocity, $T = Mg$.

**Motion of Connected Particles (with unstretchable strings)**

$T_1 = M_1 a$
$T_2 - T_1 = M_2 a$
$F - T_2 = M_3 a$

$$\Rightarrow a = \frac{F}{M_1 + M_2 + M_3}$$
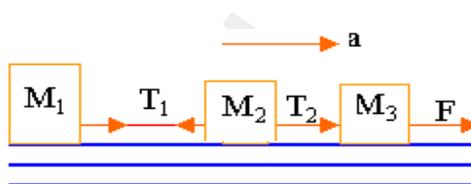


The above result can be obtained quickly by taking the three masses as one system if we are interested in acceleration only. Now tensions become internal forces and need not be considered. The only horizontal force in horizontal direction is F. Hence, $F = (M_1 + M_2 + M_3)a$ and a can be found.

**Inclined Plane (Without Friction)**

Natural acceleration down the plane = g sin θ
Driving force for acceleration 'a' up the plane, F = m (a + g sinθ)
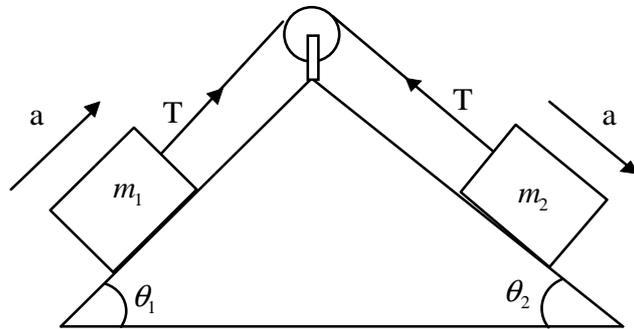and for an acceleration a down the plane, F = m (a – g sinθ).
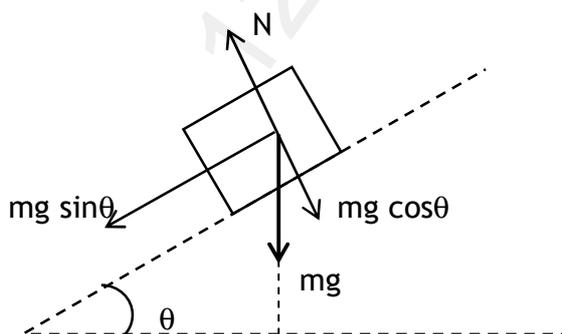
**Pulley and Inclined Plane (Without Friction)**

Using: $m_2 g \sin\theta_2 - T = m_2 a$,

$T - m_1 g \sin\theta_1 = m_1 a$.

Solving for $a = \dfrac{m_2 g \sin\theta_2 - m_1 g \sin\theta_1}{m_1 + m_2}$



**Friction**

When you try to push a table kept on the floor, you find it difficult to move. This is because of static friction.

When you slide a block on the floor, it stops on its own. This is due to kinetic friction. When a wheel rolls on the floor, it stops on its own. This is due to rolling friction. Static friction is a self-adjusting force and takes a maximum value. In general,

$$f_s \leq \mu_s N$$

Where $f_s = $ Static Frictional Force

$\mu_s = $ Coeff. of static friction . It depends on surfaces in contact.
N = Normal reaction

Kinetic friction is given by,

$$f_k = \mu_k N$$

Where $f_k = $ Static Frictional Force

$\mu_k = $ Coeff. of kinetic friction .  It depends on surfaces in contact.
N = Normal reaction

Note: Generally, it may be easier to apply kinetic friction concept while solving problems. But, one needs to be more careful while applying static friction concept, as it is a self-adjusting force. While solving problems the two coefficients of friction mentioned above are generally taken equal, however coeff. of static friction is generally greater than the coeff. of kinetic friction.

Rolling friction is due to the deformation taking place in the rolling wheel at the contact and its mathematics is beyond the scope of current discussion.

**Inclined Plane (with friction)**

For body moving down the plane replace "gsinθ" by " (gsinθ - μg cosθ) and for body moving up the plane replace "gsinθ" by "(gsinθ + μgcosθ") in the above case of frictionless inclined plane.
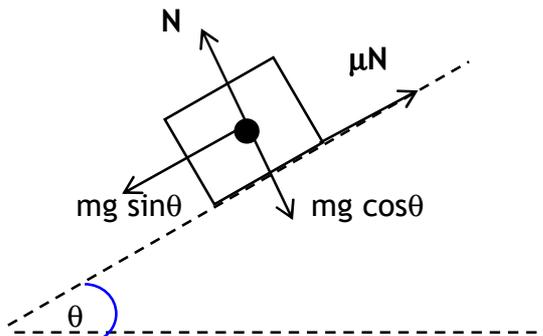


SOLVED EXAMPLES

**Example 1**

A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination $\theta$ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(A) at $\theta$ = 30°, the block will start sliding down the plane
(B) the block will remain at rest on the plane up to certain $\theta$ and then it will topple
(C) at $\theta$ = 60°, the block will start sliding down the plane and continue to do so at higher angles
(D) at $\theta$ = 60°, the block will start sliding down the plane and further increasing $\theta$, it will topple at certain $\theta$.

Only one option is correct.                                                                          [JEE 2009]

**Solution**

Sliding case

At limiting friction, when the block is just about to slide, $F_{net} = mg(\sin\theta - \mu\cos\theta) = 0$.
Or, $\tan\theta = \mu$ or $\theta = 60^\circ$.
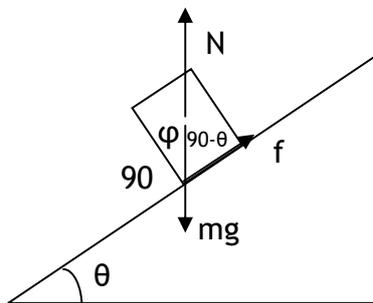
This means that sliding should begin at $\theta = 60°$.

Toppling case

mg vector in this case passes through one vertex.
This is for the minimum value of θ.

$90 - \theta + \varphi + 90 = 180$
$\Rightarrow \theta = \varphi$
$\Rightarrow \theta = \tan^{-1}\frac{10}{15} = \tan^{-1}\frac{2}{3}$

In this case $\theta < 60°$

Comparing the two cases, toppling should take place before sliding.

Hence, the best option is (B).

## Example 2

A piece of wire is bent in the shape of a parabola y = kx² (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(A) $\frac{a}{gk}$

(B) $\frac{a}{gk}$

(C) $\frac{a}{gk}$

(D) $\frac{a}{gk}$

[JEE 2009]

### Solution

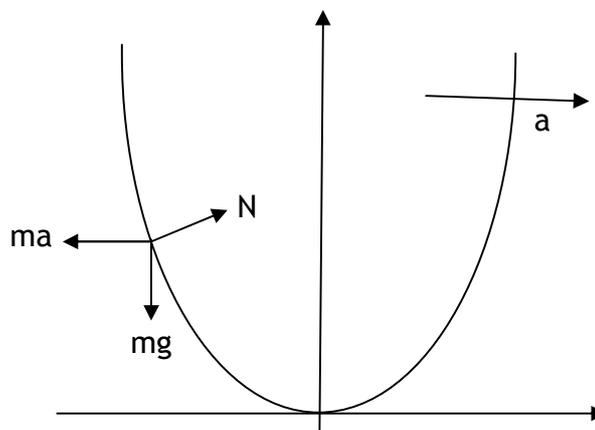Let us consider the observer in the accelerated wire frame.

Let the coordinates of the bead be (x, y).

Slope of the tangent = $\frac{dy}{dx} = 2kx$

Slope of normal = $-\frac{1}{2kx} = tan\theta$

$N\cos\theta = ma$
$N\sin\theta = mg$
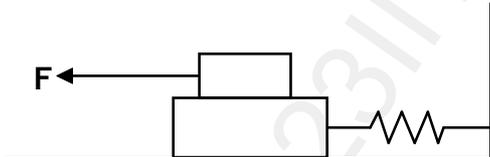
$$\tan \theta = \frac{g}{a} = -\frac{1}{2kx}$$

$$\Rightarrow x = -\frac{a}{2kg}$$

x is in the second quadrant and hence is negative.

Hence, (B).

## Example 3

There are two columns. Each option on the left column may have one or more than one match on the right column. Match the options.

| Column I | Column II |
|---|---|
| (a) Static friction<br>(b) Kinetic friction<br>(c) Rolling friction<br>(d) Frictional force acting on the top block till there is no relative motion between the blocks as top block is pulled towards left with force F  | (p) Cause of spin of a cricket ball<br>(q) Thermal energy<br>(r) Electromagnetic force<br>(s) Involved between tyres and road during hard braking in moving car<br>(t) Involved between brake shoes and rim during hard braking in moving bicycle |

**Solution**

Static friction, kinetic friction and rolling friction all are electromagnetic forces out of which only static friction does not cause thermal dissipation.

a: p,r,t

The cricket ball at the time of hitting the ground is rotating and is not rolling on the ground (rolling means rotation + translation). When a rotating ball hits the ground and particles of the ball in contact are moving in one direction, the static force acts in the opposite direction causing deviation in the path. This deviation in the path is called spin motion. Hard braking means no rotation of the wheel. The brake shoes grab the

rim of the wheel in this case to prevent the rotation.

b: q,r,s

At hard braking when no rotation is allowed to take place the tyre cannot roll, it has to skid.

c: q,r

Rolling friction involves deformation causing thermal energy.

d: p,r,t

When there is no relative motion, it is the static friction that acts.

## Example 4

STATEMENT-1
It is easier to pull a heavy object than to push it on a level ground.
**and**
STATEMENT-2
The magnitude of frictional force depends on the nature of the two surfaces in contact.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

[JEE 2008]

**Solution**

Statement-2 is correct, so is Statement-1; but Statement-1 is correct due to other reason that is the dependence of frictional force on the normal reaction. When an object is pushed then it increases the normal reaction. This increase in normal reaction increases frictional force making it difficult to push. On the other hand when an object is pulled then it decreases the normal reaction. This decrease in normal reaction decreases frictional force making it easier to pull.

Hence, (B).

## Example 5

A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$, where $A$ and $k$ are constants. The angle between the force and the momentum is

(A)  0°         (B)  30°         (C)  45°         (D)  90°

[JEE 2007]

**Solution**

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \vec{F} = A[-k\sin(kt)\hat{i} - k\cos(kt)\hat{j}]$$

$$\Rightarrow \vec{F} = -Ak[\sin(kt)\hat{i} + \cos(kt)\hat{j}]$$

Clearly, $\vec{F}.\vec{p} = 0$

Hence option (D)

## Example 6

STATEMENT-1

A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

**because**

STATEMENT-2

For every action there is an equal and opposite reaction.

(A)  Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B)  Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C)  Statement-1 is True, Statement-2 is False
(D)  Statement-1 is False, Statement-2 is True

[JEE 2007]

**Solution**

If the cloth is pulled quickly, the dishes are not dislodged from the table if the friction between the cloth and the dishes is not too high. This can be thought of due to the inertia of the dishes. So, Statement-1 is True but Statement-2, which is Newton's third law of motion and hence is also True but does not explain Statement-1. Hence, (B).

**Example 7**

The string between mass m and 2m is inextensible and light and the spring is ideal. If the string is cut find the magnitudes of accelerations of mass 2m and m
(a) g, g          (b) g, g/2
(c) g/2, g        (d) g/2, g/2
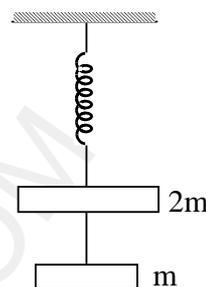
[JEE 2006 Based]

**Solution**

Initially the spring force Kx = 3 mg  (before the string is cut)
Just after the string is cut, for block 2m

$Kx - 2mg = 2ma_1$
$\Rightarrow 3mg - 2mg = 2ma_1$
$\Rightarrow a_1 = g/2$ (upward direction)

For block of mass m,

$mg = ma_2$
or  $a_2 = g$ (downward acceleration just like freely falling body)
$\therefore$  (c)

**Example 8**

A disc is kept on a smooth horizontal plane with its plane parallel to horizontal plane. A groove is made in the disc as shown in the figure. The coefficient of friction between a mass m inside the groove and the surface of the groove is 2/5 and sin θ = 3/5. Find the acceleration of mass with respect to the frame of reference of the disc.

$a_o = 25 \text{ m/s}^2$
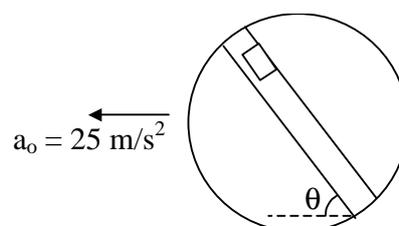
[JEE 2006 Based]

**Solution**

There are two types of frictional forces acting here, one from the bottom and the other from the side walls.
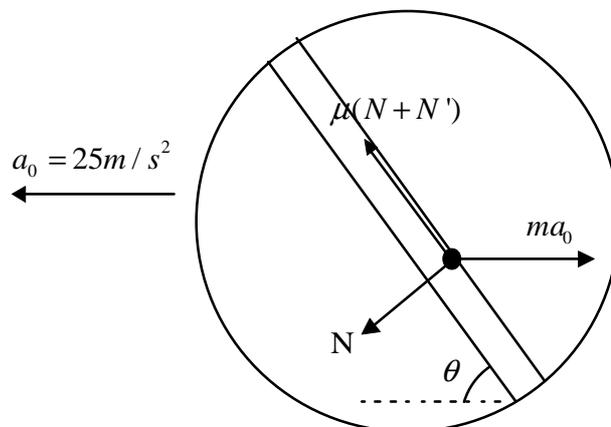


$a_0 = 25 m/s^2$

Mg=N

$ma_0 \sin\theta = N'$

$ma_0 \cos\theta - \mu(N+N') = ma$

$\Rightarrow ma_0 \cos\theta - \mu(ma_0 \sin\theta + mg) = ma$

$\Rightarrow a = a_0 \cos\theta - \mu(a_0 \sin\theta + g)$

$= 25 \times \frac{4}{5} - \frac{2}{5}\left(25 \times \frac{3}{5} + 10\right)$

$= 10 \text{ m/s}^2$

**Example 9**

Two blocks *A* and *B* of equal masses are released from an inclined plane of inclination 45° at *t* = 0 from rest. The coefficient of kinetic friction between the block *A* and the inclined plane is 0.2 while it is 0.3 for block *B*. Initially, the block *A* is $\sqrt{2}$ m behind the block *B*. When and where their front faces will come in a line. Take *g* = 10 m/s².
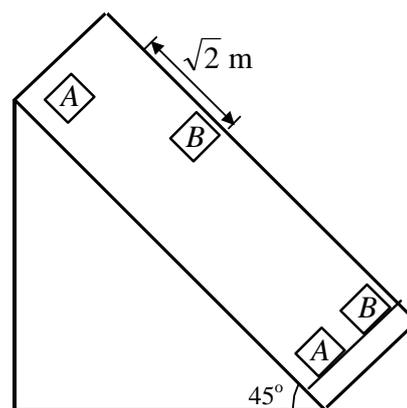


[JEE 2004 Based]

**Solution**

$a_{rel} = a_A - a_B = g(\sin\theta - \mu_A \cos\theta) - g(\sin\theta - \mu_B \cos\theta) = \dfrac{1}{\sqrt{2}}$ m/s²

$S_{rel} = \dfrac{1}{2} a_{rel} t^2$

$\Rightarrow \quad \sqrt{2} = \dfrac{1}{2} \dfrac{1}{\sqrt{2}} . t^2$

$\Rightarrow \quad t = 2$ s

Distance moved by block *A* in this time $= \dfrac{1}{2} a_A t^2 = \dfrac{1}{2} \times g(\sin\theta - \mu_A \cos\theta) t^2$

$$= 8\sqrt{2} \text{ m}$$

Therefore, the front faces of the blocks will come in a line after *A* has traveled a distance $8\sqrt{2}$ m (or *B* has traveled a distance $7\sqrt{2}$ m).

## Example 10

A block of mass $\sqrt{3}$ kg is placed on a rough horizontal surface $\mu = \dfrac{1}{2\sqrt{3}}$. A force is applied as shown in the figure. Minimum force required just to slide the block is
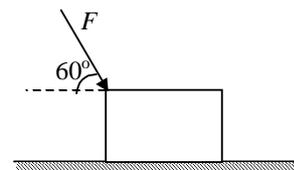
(a) 20/3 N          **(b) 20 N**
(c) 5 N              (d) 10 N

[JEE 2003 Based]

**Solution**

We have,
$$F \cos 60° = \mu N$$
$$N = F \sin 60° + mg$$
$$N = 10\sqrt{3} + \frac{\sqrt{3}}{2} F$$

And
$$\frac{F}{2} = \frac{1}{2\sqrt{3}} \left( 10\sqrt{3} + \frac{\sqrt{3}}{2} F \right)$$
$$\Rightarrow \quad F = 20 \text{N}$$

## Example 11

(a) A person sitting in a train, moving with constant velocity along a straight line, throws a ball vertically upwards - will the ball return to thrower's hand? Why?

(b) A block of mass M is suspended by a cord C from the ceiling and another cord D is attached to the bottom of the block. If you give a sudden jerk to D, it will break, but if you pull on D steadily C will break; explain.
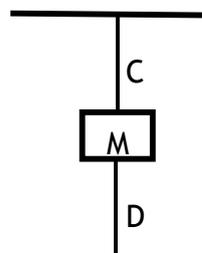
**Solution**

(a) The ball will return to thrower's hand because inertia of the horizontal velocity of ball remains equal to velocity of train.

(b) If the string D is given a jerk, the inertia of the string C is not disturbed and D breaks. But if the string D is pulled slowly, tension is transferred to C.  $T_C = T_D + Mg$ hence the string "C" breaks.

**Example 12**

(a) If you jump off a chair, you accelerate towards the earth; does the earth also accelerate towards you?

(b) A bird is sitting on the floor of a closed glass cage in the hands of a boy. If the bird starts flying (i) upwards with constant velocity (ii) upwards with acceleration (iii) downwards with acceleration, does the boy experience any change in weight of the cage?

**Solution**

(a)  Yes, the earth accelerates towards us but the acceleration is too less to realise as a=F/M where M is the mass of the earth.

(b)

(i) In case of constant velocity there is no reaction and hence boy feels no change in wt.

(ii) In case of upward acceleration, $w' = \dfrac{w}{g}(g+a)$ so the boy feels heavier.

(iii) In case of downward acceleration, $w' = \dfrac{w}{g}(g-a)$ so the boy feels lighter.

**Example 13**

(a) With what minimum acceleration can a fireman slide down a rope whose breaking strength is two third of his weight?
(b) A bird sits on a stretched telegraph wire. What is the additional tension produced in the wire in terms of its weight?

**Solution**

(a) $B = \dfrac{2}{3}mg, a = \left( g - \dfrac{B}{M} \right) = \left( g - \dfrac{2}{3}\dfrac{mg}{m} \right)$

$$a = \left( g - \frac{2}{3}g \right) = \frac{3g - 2g}{3} = \frac{g}{3}$$

(b) 2T sin θ = w, $T = \dfrac{w}{2\sin\theta}$ .

When θ → 0, T >>> w
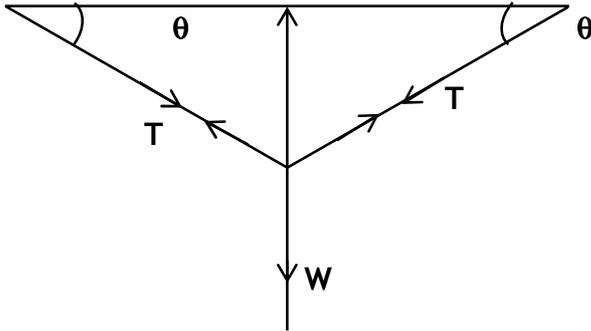


## Example 14

A man sits on a chair supported by a rope passing over a frictionless fixed pulley. The man who weighs 1000 N exerts a force of 450 N on the chair downwards while pulling on the rope. If the chair weighs 250 N and g = 10 m/s², what is the acceleration of the chair?
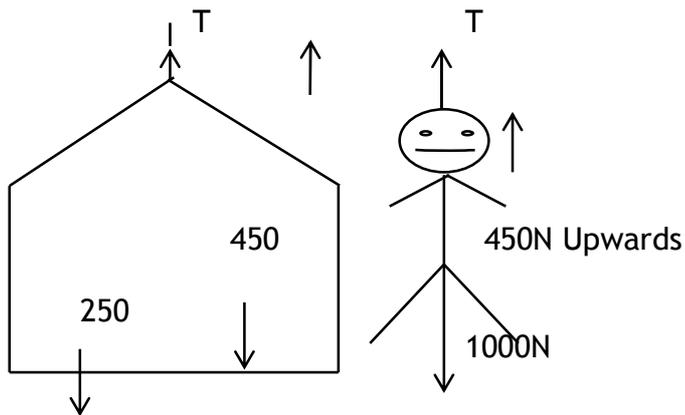
**Solution**

Mass of man = 100 kg,

Mass of chair = 25 kg.

a = ?

For chair:

T – 450 - 250 = 25 a.

For man:

T + 450 – 1000 = 100a.

a = 2m/s², T = 750 N.

T                    T

450            450N Upwards

250            1000N

## Example 15

In the given figure what should be the value of mass 'm' for which the pulley - 2 will remain stationary?



Pulley –1

m

Pulley – 2

2kg

3kg
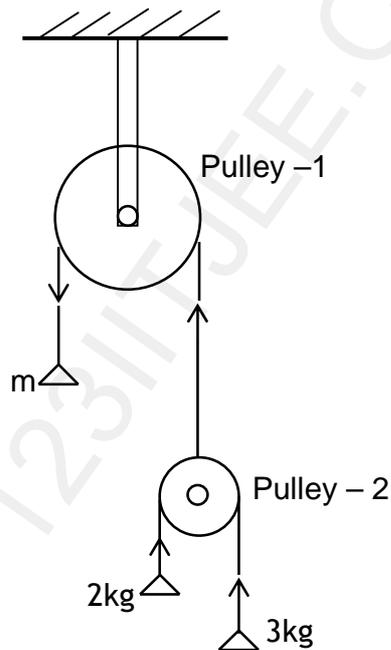
### Solution

Thrust on the lower pulley (the lower pulley and the two block system make Atwood machine and its result for tension can be used)

$$2T = \frac{2 \times 2M_1 M_2}{M_1 + M_2} \times g$$

$$= \frac{4 \times 2 \times 3}{5} \times 10 = 48N \ .$$

48N = m x 10

m = 4.8Kg.

**Example 16**

Two point masses, each of mass M, are connected by a light string of length 2 L. The whole system is kept in a smooth horizontal table. A continuous force F is applied at the midpoint of the string at right angles to the initial position of the string between the masses. Prove that the acceleration of "M" in the direction at right angles to F is

$$\frac{F}{2M} \cdot \frac{X}{\sqrt{(L^2 - X^2)}} \ .$$

Where "X" is the perpendicular distance of particles from the line of action of F. Also discuss the situation when X = L.

**Solution**

Tension on the two sides should be the same as the structure is symmetric.
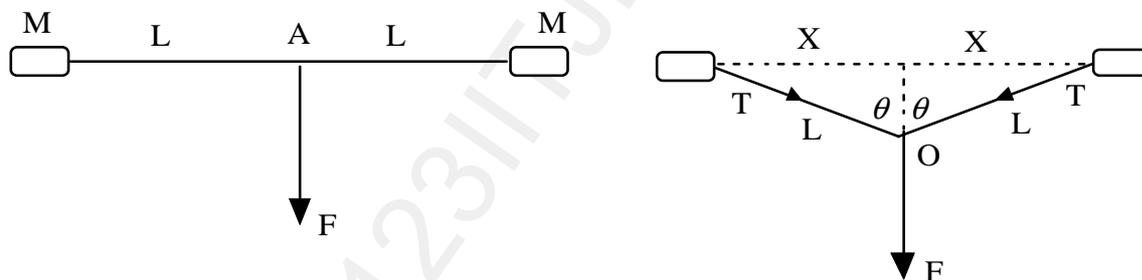


T = Ma (vertical force due to gravity has not been considered as the table is in the horizontal plane)

Also, $2T \cos \theta = F$

$$\Rightarrow T = \frac{F}{2 \cos \theta}$$

Component of acceleration perpendicular to F = $a \sin \theta$

$$a \sin \theta = \frac{T}{M} \sin \theta$$

$$= \frac{F}{2\cos\theta}\frac{\sin\theta}{M} = \frac{F\tan\theta}{2M}$$

$$= \frac{F}{2M}\frac{X}{\sqrt{L^2 - X^2}}$$

a is not defined when x = L.

## Example 17

The two blocks are connected by a heavy uniform rope of mass 4 kg. An upward force of 200 N is applied as shown in figure.

What is the acceleration of the system?
What is the tension at the top of the heavy rope?
What is the tension at the midpoint of the rope?
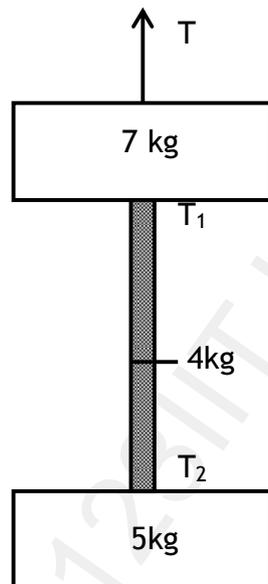


### Solution

(a) T = M(g+a)
$\Rightarrow$ 200 = (7+4+5) (9.8 + a)
$\Rightarrow$ a = (200/16) – 9.8
= 2.7 m/sec$^2$.

(b) $T_1$ = (4 + 5) (g+a)
= 9 (9.8 + 2.7)
$\Rightarrow T_1$ = 9 x 12.5 = 112.5N.

(c) T = (5 + 2) (g + a)

= 7(9.8 + 2.7)
= 87.5 N.

## Example 18

A block is kept on the floor of an elevator at rest. The elevator starts descending with an acceleration of 12 m/s$^2$. Find the displacement of the block during the first 0.2 seconds after the start.

### Hint

As the elevator is descending with an acceleration 12m/s$^2$ (>g) so the block will lose contact hence will fall freely.

## Example 19

A man is raising himself and the platform on which he stands with a uniform acceleration of 5 m/sec$^2$ by means of the rope-and-pulley arrangement as shown in Fig. A. The pulley is suspended from an overhead beam. The man has mass of 100 Kg and the platform is 50 Kg. Assume that the pulley and rope are massless and move without friction, and neglect any tilting affects of the platform. Assume g = 10 m/sec$^2$.

(i) What are the tensions in the ropes A, B, and C?

(ii) What is the force of contact exerted on the man by the platform?



Fig. A
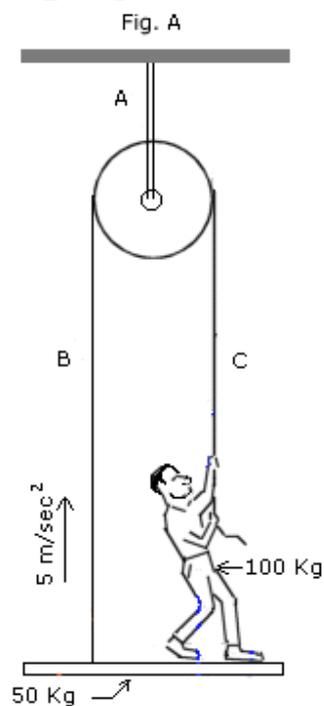
**Solution:**

Let the tension in rope A be $T_a$ and in the sections B and C of the lower rope be $T_b$ and $T_c$. As the ropes are massless, the tension at all the points in the lower rope would be the same, and

therefore $T_b$ is equal to $T_c$. Since the massless pulley is in translational equilibrium,
**$T_a = T_b + T_c$**

Now, consider the man and the platform as two isolated systems and identify the forces acting on each of the object.

Fig. B

The forces on the man are shown in blue color and that on the platform in orange color. Beginners should see 'Tips' headline at the bottom to learn about **Free-body diagram.**

The forces acting on the man are:

(a) 100g - downward weight
(b)  R - upward contact force exerted by the platform on the man.
(c)  $T_c$ - upward contact force exerted by the rope on the man.

The platform and the man, both are accelerating up at 5 m/sec², and therefore the equation of motion for the man could be written as:

$$T_c + R - 100g = 100 \times 5$$

or, $$T_c + R = 100 \times 10 + 100 \times 5$$

or, $$T_c + R = 1500 \text{ N} \qquad \dots (i)$$

The forces acting on the platform are:

(a) 50g - downward weight
(b) R - downward contact force of the man on the platform.
(c) $T_b$ - pull of the rope on the platform.
The equation of motion for the platform is:

$$T_b - R - 50g = 50 \times 5$$

or, $$T_b - R = 50 \times 10 + 50 \times 5$$

or, $$T_b - R = 750 \text{ N} \qquad \dots (ii)$$

Eliminating R from the above two equations,

$$T_b + T_c = 2250 \text{ N}$$

or, **$T_a = 2250$ N**

Since $T_b$ and $T_c$ are equal,

**$T_b = T_c = 1125$ N**

And from (ii), you get

$$R = T_b - 750$$

or. $$R = 1125 - 750$$

or, **R = 375 N**

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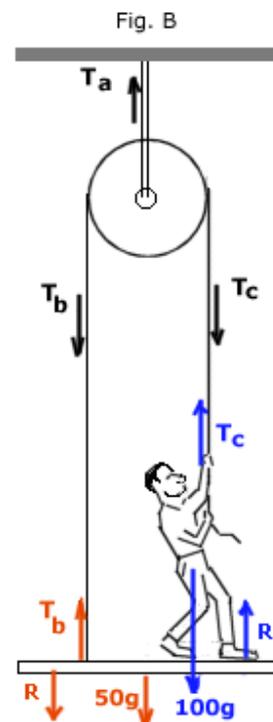
**You may now try this:**

Suppose the man is standing on a light weighing machine kept on the platform and he applies just enough pull on the rope so as to keep the platform at rest. What would be the weight of the man shown by the weighing machine in this situation?

**TIPS**

---

Newton's laws refer to a particle and relate the forces acting on the particle to its mass and to its acceleration. But before writing any equation from Newton's law, you should be careful about which particle you are considering. The laws are applicable to an extended rigid body too which is nothing but collection of a large number of particles.

Follow the steps given below in writing the equations:

## Step 1: Select the body

The first step is to decide the body on which the laws of motion are to be applied. The body may be a single particle, an extended body like a block, a combination of two blocks - one kept over another or connected by a string. The only condition is that **all the parts of the body or system must have the same acceleration.**

## Step 2: Identify the forces

Once the system is decided, list down all the forces acting on the system due to all the objects in the environment such as inclined planes, strings, springs etc. However, any force applied by the system shouldn't be included in the list. You should also be clear about the nature and direction of these forces.

## Step 3: Make a Free-body diagram (FBD)

Make a separate diagram representing the body by a point and draw vectors representing the forces acting on the body with this point as the common origin.

This is called a free-body diagram of the body.

Look at the adjoining free-body diagrams for the platform and the man. Note that the force applied by the man on the rope hasn't been included in the FBD.



Once you get enough practice, you'd be able to identify and draw forces in the main diagram itself instead of making a separate one as it has been done in Fig.B on the main solution page.
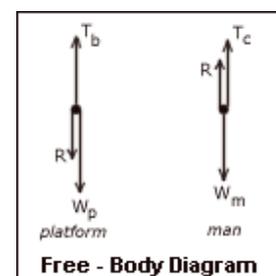
## Step 4: Select axes and Write equations

If the forces are coplanar (you'd mostly deal with forces in a plane), only two mutually perpendicular axes say X and Y in the plane of the forces need be taken. Choose X-axis along the direction in which the body is known or likely to have the acceleration. The Y-axis obviously shall be perpendicular to it. If the body is in equilibrium, any mutually perpendicular directions may be chosen as axes.

Now, write the components of the forces along the X-axis and equate their sum to the product of mass and acceleration of the body if its motion is along this axis. Also, write the components of the forces along the Y-axis and equate their sum to zero if the body is in equilibrium along this axis.

## Example 20

Two masses m and 2m are connected by a massless string which passes over a frictionless pulley as shown in the figure. The masses are initially held with equal lengths of the strings on either side of the pulley. Find the velocity of masses at the instant lighter mass moves up a distance of 6.54m. The string is suddenly cut at that instant. Calculate the time taken by each mass to reach the ground.     [JEE 1977]
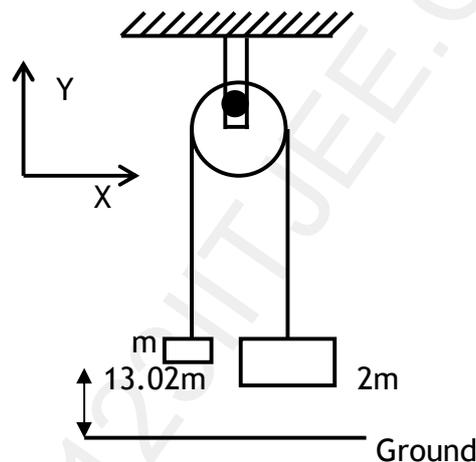


**Solution**

**Step 1.** As the masses are released mass 2 m will start to move down with acceleration a, while mass m will rise with same acceleration.

Hence,



**FBDs**
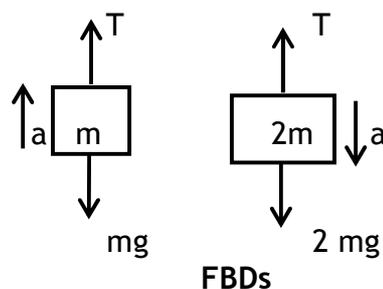
T - m g = m a
2 m g - T = 2 m g a
That gives 'a' = g/3, T = (4/3) m g

**Step 2.** Both masses are moving with constant acceleration. Their velocity after moving a distance equal to 6.54 m is given by

$$v^2 = 2aS \Rightarrow v = \sqrt{2 \times \frac{10}{3} \times 6.54} = 6.6 ms^{-1}$$

**Step 3.** The string is now cut. The masses will then move vertically freely under acceleration due to gravity and ultimately hit the ground.

For mass m,

$v_{oy}$ = +6.6 ms$^{-1}$ (upward),

$y_o$ = 13.02 + 6.54 = 19.56 m

y = 0, a = -10 ms$^{-2}$.

Hence, from equation
$y = y_o + v_{oy} t + \frac{1}{2} at^2$

We have 5 t$^2$ - 6.6 t - 19.56 = 0

or t $= \dfrac{6.6 + \sqrt{6.6^2 + 4 \times 5 \times 19.56}}{10} \approx 2.74 s$

For mass 2 m,

$v_{oy}$ = - 6.6 ms$^{-1}$ (downward),

$y_o$ = 13.02 - 6.54 = 6.48 m,

y = 0, a = -10 ms$^{-2}$. Hence

5 t$^2$ + 6.6 t - 6.48 = 0

$or \qquad t = \dfrac{6.6 + \sqrt{6.6^2 + 4 \times 5 \times 6.48}}{10} = 0.65 s$

## Example 21

Two bricks each having a mass of 0.2 kg tied at the ends of a light flexible cord passing over a small frictionless pulley as shown in the figure below. A 0.1 kg block is placed on the right block, and then removed after 2 s.
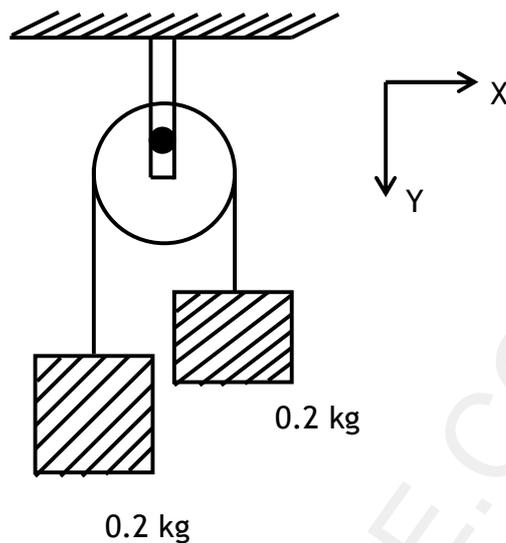


(a) How far will each block move in the first second after the 0.1 kg block is removed?
(b) What was the tension in the cord, before 0.1 kg block was removed and after it was removed?
(take g = 10 ms$^{-2}$)

**Solution**

When 0.1 kg mass is placed on the right, the left mass will move up with an acceleration 'a' and right mass will move down with the acceleration 'a'.
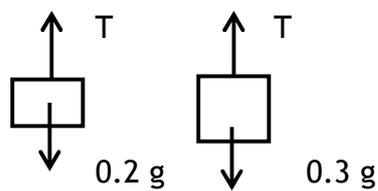


(Left side)     (Right side)

T - 0.2 g = 0.2 a --------(1)
0.3 g - T = 0.3 a ---------(2)

We get 'a' as $\dfrac{(0.1)g}{0.5} = \dfrac{0.1 \times 10}{0.5} = 2ms^{-2}$

This acceleration acts on both the masses for t = 2 s. Therefore the velocity of each mass at t = 2 s, starting from rest under constant acceleration a, is

v = at = 2 ×2 = 4m/s

At this time the 0.1 kg mass is removed, masses on two sides become equal. Note that now there is no acceleration in the system and therefore masses will continue to move further with constant velocities of 4 ms⁻¹. The distance now moved in 1s is

y = vt = 4× 1 = 4 m

(b)

(i) tension in the cord before 0.1 kg mass is removed from (1) and (2),

T = 2.4 N

After the 0.1 kg mass is removed, there is no acceleration in the system, and since masses on both sides are 0.2 kg each, we find



T = 0.2 g
  = 0.2 ×10
  = 2 N

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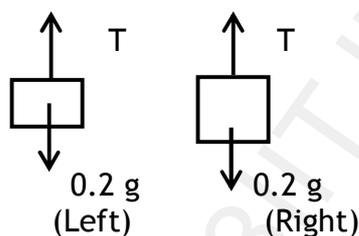
## Example 22

Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support by two inextensible wires each of length 1 m, see figure. The upper wire has negligible mass and the lower wire has a uniform linear density of 0.2 kgm⁻¹. The whole system of blocks, wires and support have an upward acceleration of 0.2 ms². (g = 9.8 ms²)

(i) Find the tension at the mid-point of the lower wire.
(ii) Find the tension at the mid-point of the upper wire. [JEE 1989]
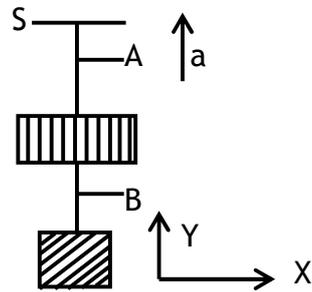
**Solution**

(a) In order to find tension $T_B$ at middle point B of the lower string, consider the FBD of lower part as shown in the figure above. Mass of this part is

M' = 1.9 + 0.1 = 2.0 kg

Hence $T_B$ - M'g = M'a,

$\Rightarrow T_B$ = M'(g + a)
   = 2.0 × (9.8 + 0.2)
   = 20 N

(b) To find tension $T_A$ at the middle point A of the upper wire, consider the next FBD with all the masses. Now the total mass is M" = 1.9 + 0.2 + 2.9 = 5.0 kg.

Hence,
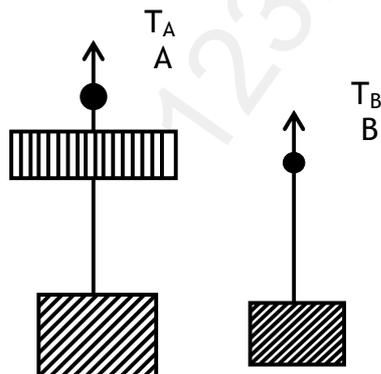


$T_A$ - M"g = M"a
or $T_A$ = M" (g + a)
      = 50 N.

**Comments**

(i) Each part of the system moves with the same acceleration (a = 0.2 ms$^{-2}$) upward, whether seen as a whole or in parts.
(ii) The tension in the lower string will be different at each point, along its length because its lower string has a mass (i.e. it is not mass less). Therefore, considering any point on this string, it has to lift the remaining lower portion of the string (along with lower block)

**Example 23**

Two blocks A and B each having a mass of 20 kg, rest on frictionless surfaces as shown in the figure below.  Assuming the pulleys to be light and frictionless, compute:

(a) the time required for block A, to move down by 2 m on the plane, starting from rest, (b) tension in the string, connecting the blocks.
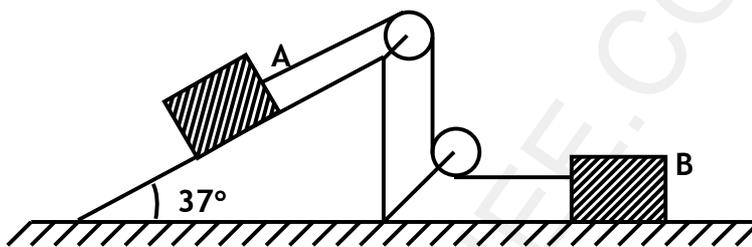


**Solution**

**Step 1**. Draw the FBDs for both the blocks. If tension in the string is T, then we have
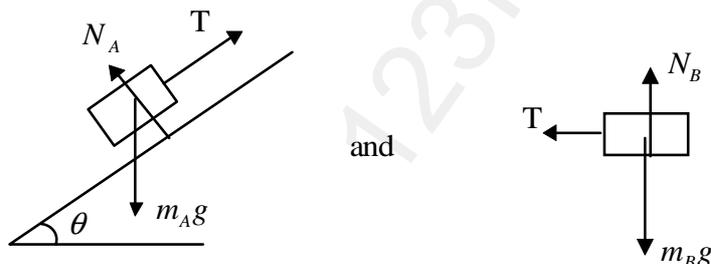


Note that m$_A$g, should better be resolved along and perpendicular to the plane, as the block A is moving along the plane.
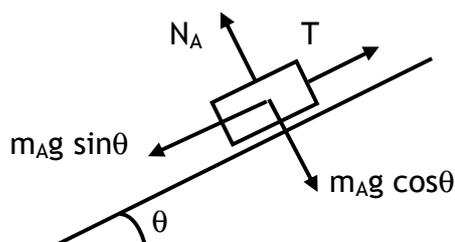
**Step 2.** From FBDs, we write the force equations;

for block A where
$N_A = m_A \, g \cos \theta = 20 \times 10 \times 0.8 = 160$ N
and $m_A \, g \sin \theta - T = m_A \, a$_____ (i)

Where 'a' is acceleration of masses of blocks A and B.

Similarly, force equations for block B are

$N_B = m_B g = 20 \times 10 = 200$ N.
and $T = m_B a$ _____(ii)

From (i) and (ii), we obtain

$$a = \frac{m_A g \sin \theta}{m_A + m_B} = \frac{20 \times 10 \times 0.6}{40} = 3 m \, s^{-2}$$

$T = m_B \, a = 20 \times 3 = 60$ N

**Step 3.** With constant acceleration a = 3 ms$^{-2}$, the block A moves down the inclined plane a distance S = 2 m in time t given by

$$S = \frac{1}{2} a t^2$$

$$or \ \ t = \sqrt{\frac{2S}{a}} = \frac{2}{\sqrt{3}} \sec onds.$$

## Example 24

Find the acceleration of each block in the figure shown below; in terms of their masses $m_1$, $m_2$ and g. Neglect any friction.
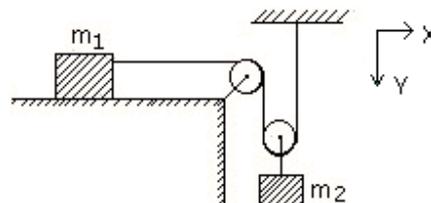
**Solution**

Let T be the tension in the string that is assumed to be mass less.
For mass $m_1$, the FBD shows that
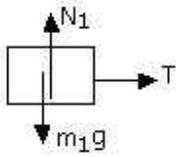
$N_1 = m_1g$

Where $N_1$ is the force applied upward by plane on the mass $m_1$. If acceleration of $m_1$ along horizontal is $a_1$, then

$T = m_1a_1$    (i)

For mass $m_2$, the FBD shows that

$m_2g - 2T = m_2a_2$    (ii)
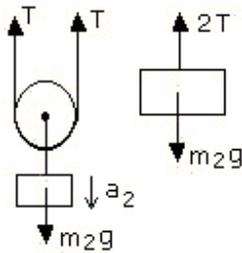


Where $a_2$ is vertical acceleration of mass $m_2$. Note that upward tension on $m_2$ is 2T applied by both sides of the string.

Constraint: Equation (i) and (ii) cannot be solved for three unknowns a1, $a_2$ and T. The third relation is provided by the constraint equation which is as follows:

If mass $m_1$ moves a distance x in time t, then mass $m_2$ moves a distance y = x/2 vertically downwards.

This can be seen from the geometry of the figure since total length of the string is constant. Hence,

$$\frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt}$$

and

$$\frac{d^2y}{dt^2} = \frac{1}{2}\frac{d^2x}{dt^2} \Rightarrow a_2 = \frac{a_1}{2} \quad (iii)$$

Thus, the acceleration of $m_1$ is twice that of $m_2$. With this input, solving (i) and (ii), we find

$$a_1 = \frac{2m_2 g}{4m_1 + m_2}$$

$$a_2 = \frac{m_2 g}{4m_1 + m_2}$$

## Example 25

Three blocks of masses $m_1$, $m_2$ and $m_3$, are placed as shown in the figure (masses $m_1$ and $m_3$ are in contact). All the surfaces are frictionless and assume the pulleys and strings to be massless. If a force F is applied on $m_1$ as shown, find its horizontal acceleration.
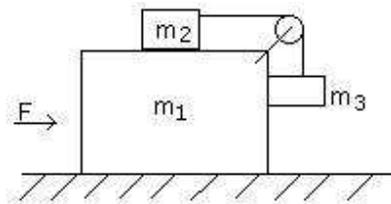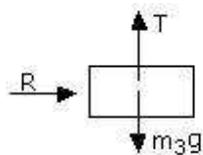
**Solution**

**Step 1.** Draw the free body diagrams of $m_1$, $m_2$, $m_3$ as shown below.

For $m_2$
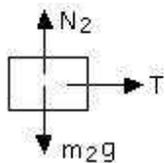
For $m_3$

Note: R is the normal force by which $m_1$ presses (or pushes) $m_3$.

For $m_1$
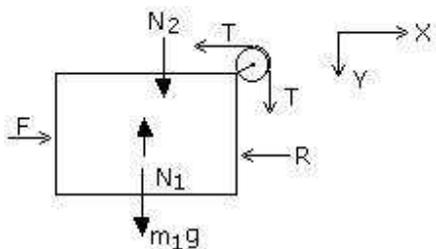
Note: $m_3$ presses on $m_1$ by force R (action-reaction). Similarly $m_2$ presses on $m_1$ by force $N_2$ (action-reaction). $N_1$ is normal force applied on $m_1$ by the table (or surface).

**Key:** Important is the fact, that pulley is fixed to mass $m_1$. The string applies tension force on the pulley (or $m_1$) in both, horizontal and vertical directions as shown above.

**Step 2.** Once FBDs are drawn let us write down equations of motion for horizontal and vertical motions,

For $m_2$,

$N_2 = m_2 g$ (No vertical motion)
$T = m_2 a_{2x}$ (towards right)
For $m_3$, $m_3 g - T = m_3 a_{3y}$ ($a_{3y}$ is vertically downwards)
$R = m_3 a_{3x}$ (towards right)

For $m_1$;

$N_2 + T + m_1 g = N_1$ (no vertical motion)
$F - R - T = m_1 a_{1x}$ (towards right).

Note: You must feel sure (by now) that we have taken a coordinate system fixed to the table with axes as shown below.



All the accelerations are with respect to this fixed inertial frame

**Step 3.** Look for constraints before we could solve the above equations. They are as follows:

(i) Since $m_1$ and $m_3$ are touching, they move together along x-axis. Hence
$a_{1x} = a_{3x}$

(ii) Suppose in time t, $m_1$ slides a distance $x_1$ on the table. In the same time, $m_2$ slides a distance $\ell$ on top of $m_1$. Hence $m_2$ moves a distance $(x_1 + \ell)$ with respect to our

coordinate system. That is

$x_2 = x_1 + \ell$

However, $\ell$ is the distance by which $m_3$ moves down, (because that is the length of by which string shifts on the pulley or $m_1$). Hence, $\ell = y_3$, the displacement of $m_3$, Thus, we get the constraint relation as

$x_2 = x_1 + y_3$
or $a_{2x} = a_{1x} + a_{3y}$
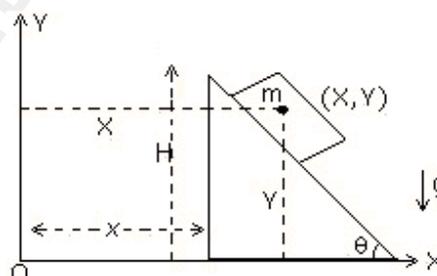(You get it by differentiating the displacements twice)

**Step 4.** Solve the given equations to find $a_{1x}$; the answer is

$$a_{1x} = \frac{F(m_2 + m_3) - m_2 m_3 g}{m_1 m_2 + (m_1 + 2m_2 + m_3)m_3}$$

$a_{3x} = a_{1x}$ and $a_{2x} = \dfrac{m_3}{m_2 + m_3}(a_{1x} + g)$.

## Example 26

A block of mass m, slides on a smooth wedge as shown in the figure. The wedge also slides on a smooth horizontal surface. Find the acceleration of the wedge.



**Solution**

The block m slides down the inclined plane and simultaneously, the wedge moves along the horizontal surface. We have inertial coordinate system (XY) fixed with the horizontal surface.

(a) Let $\vec{a}$ and $\vec{A}$ be the accelerations of the block and the wedge, respectively. In the chosen coordinate system (origin O), we write

$\vec{a} = a_x \hat{i} + a_y \hat{i}$
$\vec{A} = A\hat{i}$

Where $a_x$ and $a_y$ are x and y components of $\vec{a}$.

In order to write down the equations of motion, let us draw the FBDs of the block and the wedge.

For block, forces are as shown below. Hence,
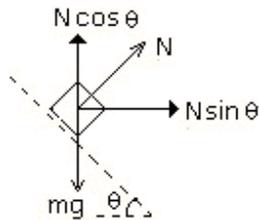


$F_x = N \sin\theta = ma_x$
and $F_y = N \cos\theta - mg = ma_y$

That gives

$$a_x = \frac{N \sin\theta}{m} - - - - (1)$$
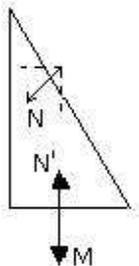
$$a_y = \frac{N \cos\theta}{m} - g - -(2)$$

For the wedge, the forces are as shown below.

Hence,

$N' = Mg + N \cos\theta$ (No vertical Motion)
and, $-N\sin\theta = MA$

That gives $A = \dfrac{-N\sin\theta}{M}$ _____(3)



(b) Constraints: Equations (1-3) are not sufficient to give values for four unknowns, viz $a_x$, $a_y$, A and N. We get one more relation from the fact that the block is constrained to move on the wedge. This gives a geometrical constraint as shown in the top figure.

Let us work it out.

Suppose (X, Y) are the coordinates of block m at time t. At the same instant let x define the position of the wedge along x-axis. From geometry, we find that

$$\tan\theta = \frac{H - y}{X - x}$$
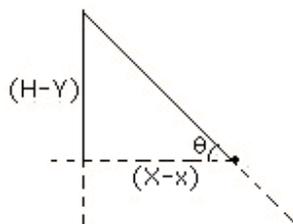


or $(X - x) \tan\theta = H - Y$

Differentiating the above relation twice with respect to t, we have
$(a_x - A) \tan\theta = -a_y$ _ _ _ _ _(4)

With the help of the above equation, we can solve for A. From (1) and (3), we get

$$a_x = \frac{-M}{m}A$$

and from (2) and (3), we have

$$a_y = -\frac{M}{m}A\cot\theta$$

Substituting for $a_x$ and $a_y$ in (4), we obtain

or $A = \dfrac{-g.\sin\theta\cos\theta}{\left(\dfrac{M}{m} + \sin^2\theta\right)}$

Negative sign means, that the wedge has acceleration towards negative x-axis.

### Example 27

A block having a mass 3 kg is initially at rest on a horizontal surface. The coefficient of static friction $\mu_s = 0.3$ between the block and the surface and $\mu_k$ is 0.25. A constant force F of 50 N, acts on the body at the angle $\theta = 37^o$. What is the acceleration of the block?
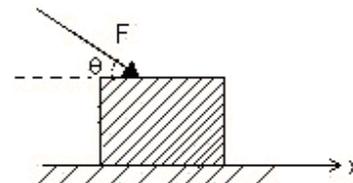
**Solution**

We have two possibilities here, the block may remain at rest, or it may accelerate towards the right. The decision hinges on whether or not the x- component of the force F has magnitude, less than or greater than the maximum static friction force.
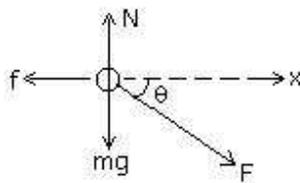
The x-component of F is
$F_x = F\cos\theta = (50\text{ N}) (0.8) = 40$ N

To find $f_{s,max}$, we first calculate the normal force N, whether or not the block accelerates horizontally, the sum of the y -component of all the forces on the block is zero.

$N - F\sin\theta - mg = 0$
or $N = F\sin\theta + mg = (50\text{ N}) (0.6) + (3\text{ kg}) (9.8 ms^{-2})$
$\qquad\qquad = 59.4$ N



The maximum static frictional force

$f_{s,max} = \mu_s N$
$\qquad = (0.3) (59.4\text{ N})$
$\qquad = 17.8$ N

This value is smaller than the x -component of F, hence the block moves. We now interpret the force f in the figure as a kinetic frictional force. This value is obtained as

$f_K = \mu_K N = (0.25) (59.4\text{ N}) = 14.8$ N

Therefore resultant force in the x-direction is

$\sum F_x = F\cos\theta - f$
$\qquad = 40\text{ N} - 14.8\text{ N}$
$\qquad = 25.2$ N

Then the acceleration 'a' of the block is

$$a = \frac{25.2N}{3kg} = 8.4 ms^{-2}$$

Think: What would happen if the magnitude of $F_x$ happened to be less than $f_{s,max}$ but larger than $f_K$?

## Example 28

In the previous example, suppose we move the block by pulling it with the help of a massless string tied to the block as shown here. What is the force F required to produce the same acceleration in the block as obtained in the last example?
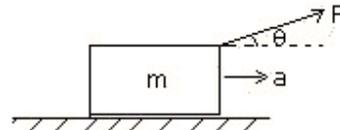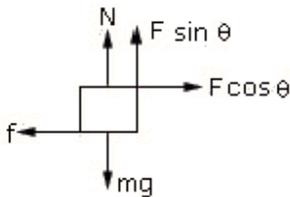
**Solution**

We are given that,

m = 3kg, $\mu_S$ = 0.3, $\mu_k$ = 0.25, $\theta$ = 37°, and a = 8.4 ms$^{-2}$

In order to determine the force F, we first draw the FBD as shown below

The equations of motion therefore, are

N + Fsin $\theta$ = mg
N = mg - Fsin $\theta$
F cos$\theta$ - f = ma

and where f = $\mu_S$ N before the start of the motion, once motion is set, f = $\mu_k$N.

Hence, force F which produces a=8.4 m/s$^2$ is given by

F cos$\theta$ - $\mu_k$ (mg -Fsin$\theta$) = ma

or F = $\dfrac{ma + \mu k mg}{\cos\theta + \mu k \sin\theta}$

$= \dfrac{3(0.4 + 0.25 \times 9.8)}{0.8 + 0.25 \times 0.6} = 34.26N$

Note that Fsinθ works out to be less than mg. Otherwise we would lift the block up in the above analysis

## Comment

It is easier to pull then to push. Only about 34 N force is required to pull than 50 N required during pushing why?

Because, when we pull at an angle, the effective normal force N by which block is pressing down on surface is reduced and consequently friction is reduced. Just the contrary happens when you are pushing.

### Example 29

As the figure shows, a block A is on a horizontal surface and a plank B is placed on top of A. The plank B is kept from moving to the left by means of the cord C. The masses of plank B and the block A are $m_B$ = 5kg and $m_A$ = 10kg respectively. The coefficient of kinetic friction $\mu_K$ between A and B is 0.2 and that between A and the horizontal surface is 0.3. A force F is applied such that it slides to the left with constant velocity. What is the magnitude of F? (take g = 10ms$^{-2}$)
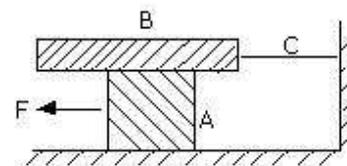


**Solution**

Let us draw the free body diagrams for A and B, and write down the equations of motion. For B: In vertical direction, we have force of gravity and normal force by block A. In horizontal direction, tension T acts towards right. Block A applies frictional force $f_A$ towards left (since B slides towards right relative to A) Hence we have:
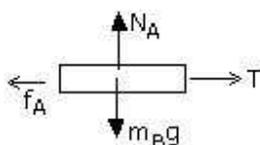


Since B remains stationary we find

$N_A = m_B g = $ (5kg) (10 ms$^{-2}$)
            = 50 N
and $T = f_A = \mu_k N_A = $ (0.2) (50 N) = 10 N

For A: The FBD for A is drawn. Note $N_A$ and $N_S$ are normal forces applied by B and surface below, respectively. Further, there are frictional forces $f_A$ and $f_s$ applied by block B and the lower surface respectively.

Now writing the equations of motion, we find

$N_S - N_A - m_A g = 0$ (No vertical motion)
$\Rightarrow N_S = N_A + m_A g = $ 50 N + 10 $\times$ 10 N
                = 150 N
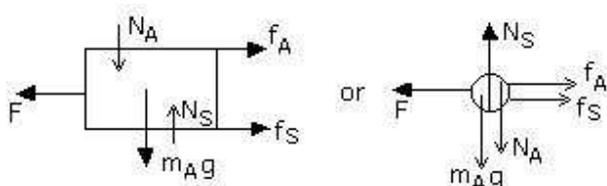


For the horizontal motion, we have
$F - f_A - f_s = 0$

Note that since A moves with constant velocity the net horizontal force must also be zero

Now, $f_s = \mu_K N_s = $ (0.3) (150 N) = 45 N.
Hence $F = f_A + f_s = $ 10 + 45 = 55N.

## Example 30

Consider a rough inclined plane whose angle of inclination $\theta$ with ground can be changed. A block of mass m is resting on the plane. Find the minimum angle $\theta = \theta_c$ when the block just begins to slide on the plane. Coefficient of (static) friction between the block and plane is $\mu$.

**Solution**

For a given angle $\theta$, the FBD of the block is
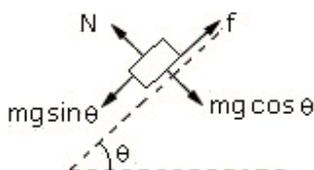
Where f is force of static friction on the block. For normal direction to the plane, we have

N = mg cos $\theta$

As $\theta$ increases, the force of gravity down the plane, mg sin $\theta$, increases. Friction force resists the slide till it attains its maximum value.

f$_{max}$ = $\mu$ N = $\mu$ mg cos $\theta$

Which decreases with $\theta$ (because cos $\theta$ decreases as $\theta$ increases).

Hence, beyond a critical value $\theta$ = $\theta_c$, the blocks starts to slide down the plane. The critical angle is the one when mg sin $\theta$ is just equal to f$_{max}$, i.e., when

mg sin $\theta_c$ = $\mu$ mg cos $\theta_c$
or tan $\theta_c$ = $\mu$ .

If $\theta$ > $\theta_c$, block will slide down. For $\theta$ < $\theta_c$ the block stays at rest on the incline.

## Example 31

Two blocks of masses m$_1$ and m$_2$ are kept in contact on a rough inclined plane as shown. Coefficient of static friction between the blocks and plane are $\mu_1$ and $\mu_2$ respectively ($\mu_1$ > $\mu_2$). If the angle of inclination $\theta$ of the plane is slowly increased, what is the minimum value of $\theta$ when the system of blocks just starts to slide down? What is the contact (normal) force between the two blocks at that instant?
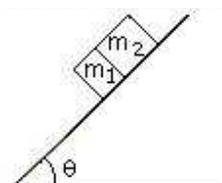


### Solution

(a) We have seen in the last example that lower block of mass m$_1$ would just tend to move by itself (in absence of m$_2$) at $\theta$ = $\theta_1$ given by tan $\theta_1$ = $\mu_1$

Similarly, block m$_2$ would start sliding by itself (in absence of m$_1$) at $\theta$ = $\theta_2$; where tan $\theta_2$ = $\mu_2$.

Since, $\mu_1 > \mu_2$, we note that $\theta_1 > \theta_2$. Hence, the condition $\mu_1 > \mu_2$ ensures that m$_1$ will not slide along leaving m$_2$ behind.

Obviously, for some angle $\theta = \theta_c$, where $\theta_2 \leq \theta_c < \theta_1$, the block as a single unit would begin to slide.

(b) We can determine $\theta_c$ by considering both the blocks as a single block of mass ($m_1 + m_2$). The FBD of the combined system would be
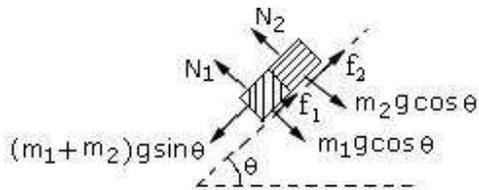


Note that it is equivalent to a single object whose base has two parts, one with coefficient of friction $\mu_1$ and other with $\mu_2$. Since the object is tending to move, frictions will act with its maximum values. Hence, for $\theta = \theta_c$, we have

($m_1 + m_2$)sin $\theta_c$ = $m_1 \theta_1$cos $\theta_c$ + $m_2 \mu_2$ cos $\theta_c$

Thus, tan $\theta_c$ = $\dfrac{m_1\mu_1 + m_2\mu_2}{m_1 + m_2}$

(check yourself that $\theta_2 < \theta_c < \theta_1$)

(c) In order to find the normal force of contact between the two block, we should consider the FBDs of two blocks separately as shown below
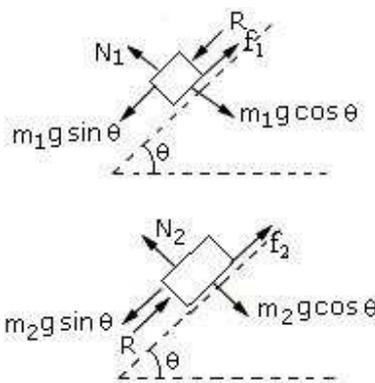


Just at the instant ($\theta = \theta_c$) the blocks are ready to move

$m_1$gsin $\theta_c$+ R = $m_1 \mu_1$gcos $\theta_c$

and $m_2$gsin $\theta_c$- R = $\mu_2 m_2$gcos $\theta_c$.

Solving for R, we get ($m_1 - m_2$)gsin $\theta_c$+ 2R = ($\mu_1 m_1 - \mu_2 m_2$)gcos $\theta_c$.

or R = $\dfrac{g}{2}$ [($\mu_1 m_1 - \mu_2 m_2$)cos $\theta_c$- ($m_1 - m_2$)sin $\theta_c$]

## Example 32

Two blocks A and B are initially at rest as shown in figure. The force $\vec{F}$ is now increased slowly from 0 to 260 N. The coefficients of friction between A and B are $\mu_k^{'}$ = 0.2, $\mu_k^{'}$ = 0.15, and that between B and lower surface are $\mu$'s = 0.15, $\mu_k$= 0.10. Masses of A and B are 35 kg and 42 kg. Plot the acceleration of both masses as a function of F. Take g = 9.8 ms$^{-2}$.
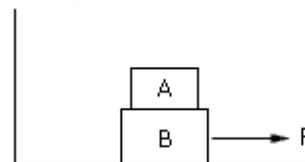


### Solution

Before we go on to solve this problem in detail, let us get a feel of the motion. Force F will move the combined blocks once it exceeds force of friction from lower surface. Once B moves, it will carry A along with itself. However, force acting on A (which moves it forward) is friction due to B and it has a maximum possible value. B cannot accelerate beyond it. Hence, once acceleration of combine block reaches beyond the value allowed for B, the two blocks will start sliding relative to each other. Let us see how it all happens.

### Phase 1.

Consider both blocks as one system. The FBD is
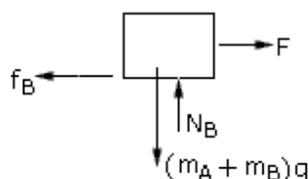


Note, $N_B$ is normal force on B from lower surface and $f_B$ is static friction between B and lower surface. We have

$N_B$ = $(m_A + m_B)$g = (35 + 42) $\times$9.8 = 754.6 N
$(f_B)_{max}$ = $\mu_s^{'}$ $N_B$ = 0.15 $\times$754.6 = 113.19 N

As long as F is less than $(f_B)_{max}$, there will be no motion.
That is, accelerations $a_A$ = $a_B$ = 0 for 0 $\leq$F $\leq$113.2 N

### Phase 2.

When F $\geq$113.2 N, the blocks start to move together. Now, kinetic friction takes over and $\mu_{1s}$ is replaced by $\mu_{1k}$. Hence, $f_B$ = $\mu_{1k}$ $N_B$ = 0.1 $\times$754.6 = 75.46 N
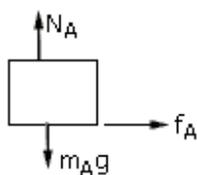
Now,

$$a_A = a_B = \frac{F - f_B}{m_1 + m_2} = \left(\frac{1}{77}\right)F - \left(\frac{75.46}{77}\right)$$

This is equation of straight line between $a_A$ (or $a_B$) and F. Just when F = 113.19 N, we get

$$a_A = a_B = \frac{113.19 - 75.46}{77} = 0.49\, m\, s^{-2}$$

**Phase 3.**

Phase 2 shows that accelerations $a_A$ (= $a_B$) would continue to increase as F increases. However, if we consider the motion of block A alone, its FBD looks as



Where $N_A$ is normal force on A by B; $f_A$ is frictional force applied by B on A. Note that as B moves to right, A tends to slide to left relative to B and hence friction $f_A$ on A acts towards right. In turn, block A applies equal force of friction $f_A$ on B towards left (this is reaction by A which resists B from moving to right), Now, for motion of A, we have

$N_A = m_A g = 35 \times 9.8 = 343$ N
and $(f_A)_{max} = \mu_s N_A = 0.2 \times 343 = 68.6$ N

That is, the maximum horizontal force which B can transfer to A is 68.6 N. Thus, according to second law applied to motion of A alone, the maximum acceleration that A can have is

$$(a_A)_{max} = \frac{(f_A)\max}{m_A} = \mu_s g = 1.96 \text{ ms}^{-2}.$$

As long as $a_B = a_A = 1.96$ ms$^{-2}$, there is no slipping between blocks B and A. They can move together. Corresponding value of F is given by

$$a_A = a_B = \frac{1}{77}F - \frac{75.46}{77}$$
or F = 1.96 $\times$ 77 + 75.46 = 226.38 N

If, 113.19 $\leq$ F $\leq$ 226.38, both the blocks move together with same acceleration given by

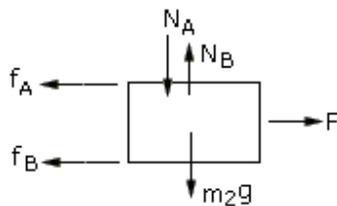a $= \dfrac{F}{m_1 + m_2} = \dfrac{F}{77} ms^{-2}$

This is the end of phase 3.

Phase 4.

For F > 226.38 N, slipping between A and B starts.

The moment A slides on B, $\mu_s$ is replace by $\mu_k$. The acceleration of A changes to $a_A$ = $\mu_k g = 0.15 \times 9.8 = 1.47 ms^{-2}$.

The frictional force between A and B becomes $f_A$ = $\mu_k M_A g = 0.15 \times 343 = 51.45$

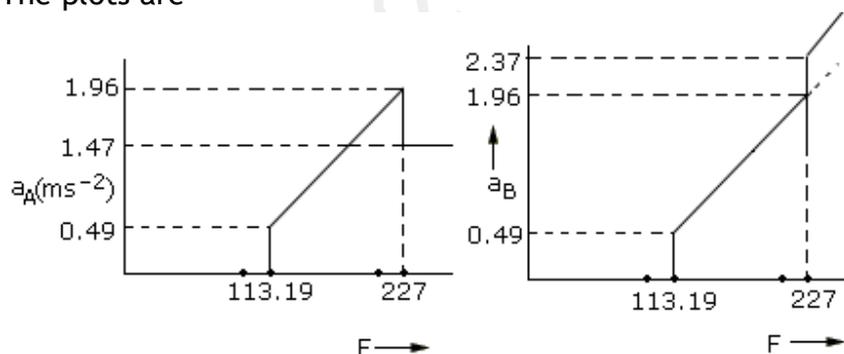Now, so long as A is sitting over B, the motion of B is given by (for F > 226.38 N):



$N_A + m_2g = N_B$
and $F - f_A - f_B = m_B a_B$
solving for $a_B$, we get $a_B = \dfrac{F}{42} - \dfrac{(51.45 + 75.46)}{42} = \dfrac{F}{42}$ - 3.02 (ms$^{-2}$).
At F = 226.38, we have $a_B$ = 2.37 ms$^{-2}$

The plots are

## Example 33

Consider the figure. Pulley and string are massless. There is no friction between the block C and the horizontal surface of the table.
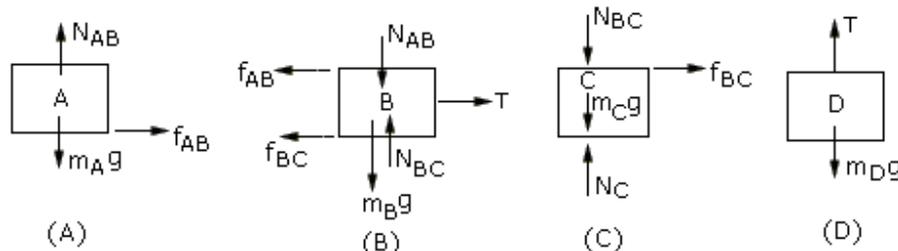
(a) Find the maximum mass $m_D$ of block D so that block A, B and C move without slipping over each other.
(b) If the mass of the block D is just greater than $m_D$ (from part (a)) which of the blocks A or C slip first with respect to B?
(c) For what value of $m_D$ will both A and C start slipping?

$m_A = m_B = m_C = m$

$\mu_{AB} = \mu_{BC} = \frac{1}{2}$
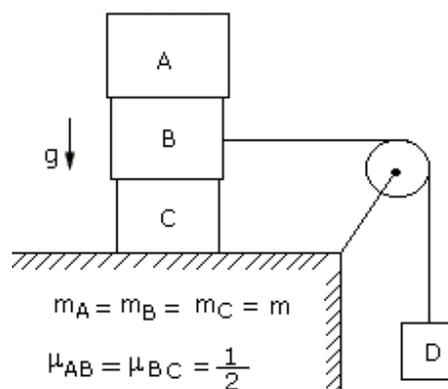
## Solution

Let us first draw the FBDs of the four blocks as shown here.

Note (i) $f_{AB}$ is friction between blocks A and B, and $f_{BC}$ is friction between blocks B and C. Block B tends to slide towards right with respect to A and C. Hence, $f_{AB}$ and $f_{BC}$ act to left on B. As reaction, $f_{AB}$ and $f_{BC}$ act to right on A and C respectively.

(ii) $N_{AB}$ is normal contact force between A and B; $N_{BC}$ is normal contact force between B and C. $N_C$ is normal force between C and table.

To begin with, the system of (A + B + C) will slide on the table.

(a) In order to analyze the situation that there is no slipping between A and B or/and between B and C, we go one by one. (i) For no slipping between A and B, let us see what is maximum acceleration possible for A.

Maximum force on A = $(f_{AB})_{max}$ = $\mu_{AB} N_{AB}$ = $\mu_{AB} m_A g$

$$= \frac{1}{2} mg.$$

Hence, maximum acceleration of A, $(a_A)_{max} = \dfrac{g}{2}$ ---- (i)

So long as B is not accelerating more than $\dfrac{g}{2}$, A will not slip on B. Thus, in order that

A and B do not slip, maximum allowed acceleration for B is $\dfrac{g}{2}$.

(ii) Similarly, for no slipping between B and C, let us see what is maximum acceleration possible for C.

Maximum force on C = $(f_{BC})max = \mu_{BC}N_{BC} = \mu_{BC}(N_{AB} + m_Bg) = \mu_{BC}(m_Ag + m_Bg)$

$$= \dfrac{1}{2} \times 2mg = mg$$

Consequently, unless acceleration of B is more than g, it will not slip on C.

(iii) combining the results of (i) and (ii), we conclude that

$(a_B)_{max} = \dfrac{g}{2}$

So that there is no slipping between A, B, and C. All the three blocks can move with

same acceleration $\dfrac{g}{2}$

Now, if $a_C = \dfrac{g}{2}$, the required frictional force $f_{BC}$ acting on C is given by

$f_{BC} = m_C \, a_C = \dfrac{mg}{2}$. (towards right)

Same force $f_{BC}$ will act on B in opposite direction (towards left). Now writing equation of motion for B, we have

$T - f_{AB} - f_{BC} = m_B \, a_B = \dfrac{mg}{2}$

or $T = f_{AB} + f_{BC} + \dfrac{mg}{2} = \dfrac{mg}{2} + \dfrac{mg}{2} + \dfrac{mg}{2} = \dfrac{3mg}{2}$

Now, for the motion of D, we find

$m_Dg - T = m_D \, a_D = m_D\dfrac{g}{2}$ (as $a_D = a_B$)

or $m_D\dfrac{g}{2} = T = 3m$

Thus, for $m_D = 3m$, all the masses will move with acceleration $\dfrac{g}{2}$ without slipping between A, B, C.

(b) If $m_D > 3m$. the block A first starts slipping with respect to B.

(c) Block C will also starts slipping only if $a_B$ is more that $(a_C)_{max} = g$. For the limiting case that $a_B = g$ we find from equation of motion for B that

$T - (f_{AB})_{max} - (f_{BC})_{max} = m_B a_B = mg$

$T = \dfrac{mg}{2} + mg + mg = \dfrac{5}{2} mg.$

On the other hand or from equation of motion for D, we find
$m_D g - T = m_D a_D = m_D a_B$ (as $a_D = a_B$)
or $T = m_D(g - a_B)$

Thus for $a_B = g$, $T = 0$; if $a_B > g$, T becomes negative. Thus no matter what is the value of $m_D$, it can never accelerate at g or more. That is, $a_B \geq g$ is not possible and therefore B never slips on C. Remember: Above results hold for $m_A = m_B = m_C$ and $\mu$

$_{AB} = \mu_{BC} = \dfrac{1}{2}$ (Not always!)

## Example 34

A block of mass m rests on a wedge of mass M, which, in turn, rests on a horizontal table as shown in the Fig. I below. All surfaces are frictionless. If the system starts at rest with point P of the block a distance h above the table, find:

(a) The acceleration of the wedge.
(b) The velocity of the block at the instant point P touches the table.

**Solution:**

It is important to understand that the wedge is not fixed to the table but is free to move. And, therefore, as the block slides down the inclined plane towards right, the wedge is pushed by the block and it moves to the left.

You may also look at the situation in another way -- in the horizontal direction there are no external forces acting on the system comprising of the block + wedge. Hence, the centre of the mass of this system remains stationary and when the block moves to the right, the wedge moves to the left.
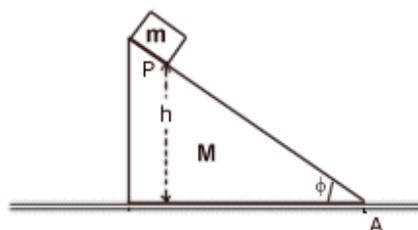
Fig. I

You'd notice that the block, as it reaches the bottom, doesn't strike point A on the table. Instead it strikes some other point A' since the wedge too has moved a distance of AA' during the descent of the block. However an observer standing on the wedge and moving with it shall notice the block to slide down the incline (path of descent making an angle ø with the horizontal) as if the wedge were stationary.

It'd therefore be convenient for you to write equations of motion for the block in a frame of reference that is attached to the wedge. But when you do so, don't forget to include the *pseudo-forces* in addition to the 'real forces' acting on the block. The motion of the wedge, nevertheless, could be considered in an inertial frame of reference attached to the ground.

Let's say the acceleration of the wedge is **a** (w.r.t. ground and acting towards left) and that for the block it is **a'** (w.r.t. to the wedge and acting in a direction down the inclined plane). Also assume that velocity of the block when it reaches the bottom is **v** (w.r.t. to the wedge). The free-body diagrams for the block and wedge are given in the Fig. II below:
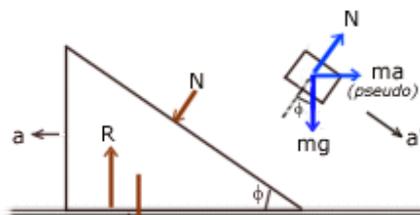


Fig. II

The forces acting on the block are:

(a) mg - weight (downwards)
(b) N - contact force due to wedge (normal to the plane)
(c) ma - pseudo force (towards right as the frame of reference is accelerating towards left)

The forces acting on the wedge are:

(a) Mg - weight (downwards)
(b) N - contact force due to the block (normal to the plane)
(c) R - contact force due to the table (Upwards)

Considering the forces acting on the block, we can write:

$$N + ma\cos\phi = mg\cos\phi \qquad ......(i)$$

And         $$ma' = mg\sin\phi + ma\cos\phi$$

or,         $$a' = g\sin\phi + a\cos\phi \qquad ......(ii)$$

Considering the forces acting on the wedge we can write:

$$N \cos\emptyset + Mg = R \qquad ......(iii)$$

And $\qquad N \sin\emptyset = Ma$

or, $\qquad N = Ma/\sin\emptyset \qquad ......(iv)$

Putting the above value of N in eq.(i), you get,

$$Ma/\sin\emptyset + ma \sin\emptyset = mg \cos\emptyset$$

or, $\qquad Ma + ma \sin^2\emptyset = mg \sin\emptyset \cos\emptyset$

or, $\qquad a(M + m \sin^2\emptyset) = mg \sin\emptyset \cos\emptyset$

or, $\qquad \mathbf{a = (mg\ sin\emptyset\ cos\emptyset)/(M + msin^2\emptyset)}$

Putting the value of a in (ii) you get,

$$a' = g \sin\emptyset + (mg \sin\emptyset \cos^2\emptyset) / (M + m \sin^2\emptyset)$$

or, $\qquad \mathbf{a' = (M + m)g\ sin\emptyset/(M + msin^2\emptyset)}$

The distance covered by the block is $h/\sin\emptyset$

hence, $\qquad v^2 = u^2 + 2a'h/\sin\emptyset$

or, $\qquad v^2 = 0 + 2a'h/\sin\emptyset$

or, $\qquad v^2 = 2[\{(M + m)g \sin\emptyset\}/(M + m\sin^2\emptyset)][h/\sin\emptyset]$

Or $\qquad \mathbf{v = [\{M + m)gh\}/(M + msin^2\emptyset)]^{½}}$

---

**You may like to try this:**

What is the displacement of the wedge on the table during the time the block descends to the bottom?

Distance moved by the wedge = $h\ m\ cot\emptyset\ / (M + m)$

You perhaps got the answer by first finding out the time taken by the block to slide down the incline and the acceleration of the wedge and then applying the laws of motion.

But there is another easier method to find the distance moved by the wedge - by examining if the 'centre of mass' of the system (block + wedge) has moved.

Taking the block and wedge together as a system, there is no external horizontal force acting on it and therefore the centre of mass of this system, which was at rest initially, will remain at the same position.

Just keep this in mind for the present. Once you finish the topic on 'centre of mass', come back to this problem and apply the concept to solve it differently.

## Example 35

A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body slide by applying the minimum possible force F. Find the magnitude of F and the direction in which it should be applied? [JEE 1987]
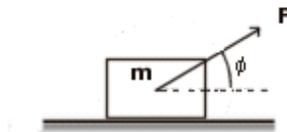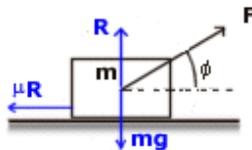


Fig. A

**Solution:**

Let the applied force F be at angle ø with the horizontal.



Fig. B

For vertical equilibrium,

$$R + F \sin\phi = mg$$

or,        $R = ( mg - F \sin\phi)$        ....(i)

For horizontal equilibrium i.e. when the block is just about to slide,

$$F \cos\phi = \mu R$$        ....(ii)

Substituting for R,

$$F \cos\phi = \mu( mg - F \sin\phi )$$

or,        $F = \mu mg / ( \cos\phi + \mu\sin\phi )$

or,        $F = \mu mg / \{ \mu^2 + 1 \cos(\phi - \delta)\}^{1/2}$

where,        $\cos\delta = 1/(1 + \mu^2)^{1/2}$

and,        $\sin\delta = \mu/(1 + \mu^2)^{1/2}$

therefore,  $F_{min} = \mu mg / ( \mu^2 + 1 )^{\frac{1}{2}}$

when,  $\phi = \tilde{o} = \tan^{-1}\mu$

## Example 36

A small block of mass m slides down the vertical side of a large rectangular block of mass M. The blocks m and M are interconnected with a string passing over a system of pulleys as shown in the diagram Fig. A. The string and pulleys are light and smooth. The coefficient of friction between the two blocks is $\mu_1$ and that between the bigger block and the ground is $\mu_2$.

Find the acceleration of the block of mass M?

**Solution:**
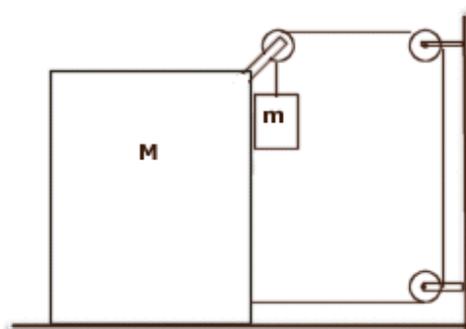


Fig. A

This one is a little tricky problem. To find the solution. not only you need to identify the forces acting on the two blocks but also figure out the relative motion of the two as well.

Do you notice any relationship between the motion of block M and m?

As the string is inextensible, whenever the lower end of the string (connected to block M) displaces by 'x' units to the right, the other end of the string (connected to block m) lowers down by '2x' units. In other words, displacement of block m by '2x' units in the downward direction, causes displacement of only 'x' units of block M to the right. By the same argument, the instantaneous velocity and acceleration of block m in vertically-downward direction is twice that of the velocity and acceleration of block M in the horizontal-right direction.

Therefore, if the acceleration of block M is **a** in the horizontal-right direction then the acceleration of block m is **2a** in the vertically-downward direction.

Also, block m is always in contact with the larger block M as far as motion in the horizontal direction is concerned. And, therefore, the acceleration of block m in the horizontal direction is also **a** - same as that of the block M.

Having defined the relationship between the acceleration of the two blocks, let's now proceed to identify the forces acting on the blocks M & m separately.
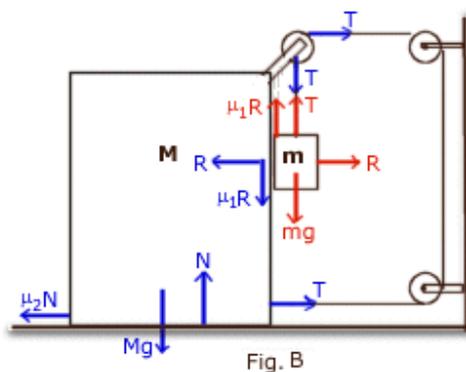
Fig. B

Look at Fig. B indicating the forces - the blue colored are the forces on block M and the orange ones act on block m.

**Motion of m:**

The acceleration is **a** in the **horizontal** direction and **2a** in **vertical** direction. The forces on m are:

(a)  mg - weight downwards
(b)  R - contact force by block M towards right
(c)  $\mu_1 R$ - frictional force upwards
(d)  T - upwards pull of the string

In the horizontal direction, the equation is:

$$R = ma \qquad \qquad .....(i)$$

In the vertical direction, the equation is:

$$mg - T - \mu_1 R = m(2a)$$

or, $\qquad T = mg - ma(2 + \mu_1) \qquad .....(ii)$

**Motion of M:**

The acceleration is **a** in the **horizontal** direction. The forces on M are :

(a)  Mg - weight downwards
(b)  R - contact force by block m towards left
(c)  $\mu_1 R$ - frictional force downwards
(d)  T - pull of the string (lower end) to the right
(e)  N - contact force by ground upwards
(f)  $\mu_2 N$ - frictional force to the left
(g)  T - pull on the pulley(fixed to M) to the right
(h)  T - pull on the pulley(fixed to M) downwards

The equation of motion for vertical equilibrium is:

$$N = Mg + T + \mu_1 R$$

or, $\qquad N = Mg + T + \mu_1 ma \qquad .....(iii)$

In the horizontal direction, the equation is:

$$2T - R - \mu_2 N = Ma \qquad .....(iv)$$

Putting values of T & N from eqns. (ii) and (iii),

2T – ma – $\mu_2$(Mg + T +$\mu_1$ma) = Ma

or,     (2 – $\mu_2$)[mg – ma(2 + $\mu_1$)] –ma – $\mu_2$(Mg + $\mu_1$ma) = Ma

or,     2mg – $\mu_2$g(m + M) =  5ma + 2ma$\mu_1$ – 2ma$\mu_2$ + Ma

or,     a[M + m{5 +2($\mu_1$ – $\mu_2$)}] = g[2m – $\mu_2$(m + M)]

or,     **a =  $\dfrac{g[2m - \mu_2(m + M)]}{[M + m\{5 + 2(\mu_1 - \mu_2)\}]}$**

## Example 37

What is the minimum acceleration with which bar A should be shifted horizontally to keep the bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal and the coefficient of friction between the bar and the bodies equal to $\mu$. The masses of the pulley and the threads are negligible while the friction in the pulley is absent. See in fig.
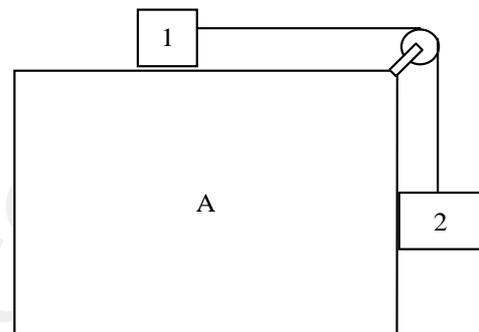
## Solution
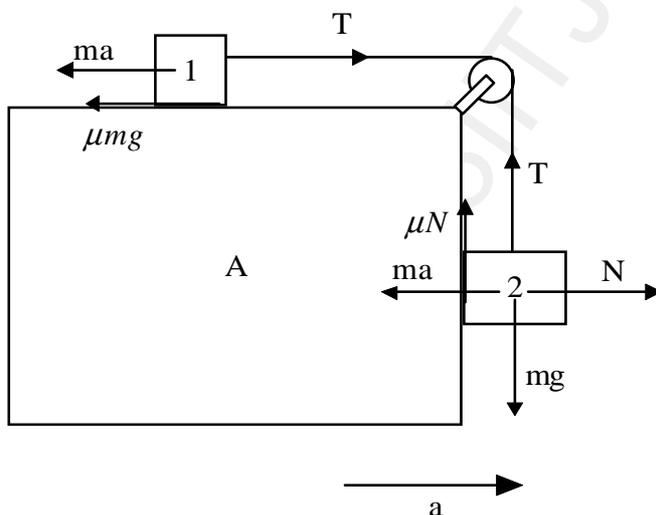


Let us place the observer on A. Since we have non-inertial frame we have pseudo forces.

For body '1' we have,

$T = ma + \mu mg$ ------(1)

For body '2' we have,

$N = ma$
$mg - T - \mu ma = 0$
$\therefore mg = T + \mu ma$ -------(2)

From (1) and (2) $a_{min} = g\left(\dfrac{1-\mu}{1+\mu}\right)$.

## Example 38

Find the accelerations $a_2$, $a_3$, $a_5$ of the three blocks shown in figure if a horizontal force of 40 N is applied on (a) 5 kg block (b) 3 kg block (c) 2 kg block.
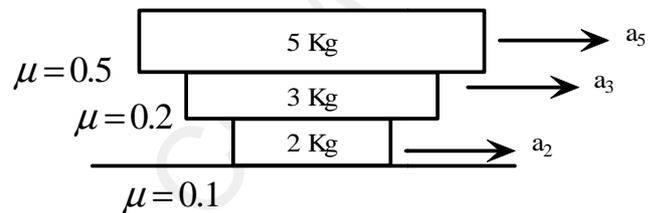


**Solution**

The free body diagram and acceleration against each diagram is shown below.

(a)

| | |
|---|---|
|  | $\therefore a_5 = \dfrac{15}{5} = 3$ m/s$^2$. |
|  | $\therefore a_3 = \dfrac{25-16}{3} = 3$m/s$^2$. |
|  | $\therefore a_2 = \dfrac{16-10}{2} = 3$m/s$^2$. |

(b)

| | |
|---|---|
|  | as 40 < 25 + 16 <br> $\therefore a_5 = a_3 = a_2 = \dfrac{40-10}{10} = 3$ m/s$^2$. |

(c)
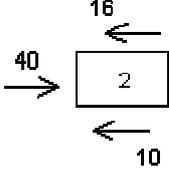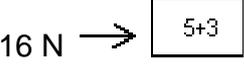
| | |
|---|---|
|  | $a_2 = \dfrac{14}{2} = 7$ m/s$^2$. |
| 16 N $\rightarrow$ [5+3] | $a_5 = a_3 = \dfrac{16}{8} = 2$ m/s$^2$. |

Blocks 5 kg and 3 kg will move together as 16 N does not overcome the static friction.

Q 4. All the surfaces shown in figure are assumed to be fricitionless. The block of mass m slides on the prism which in turn slides back-ward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.
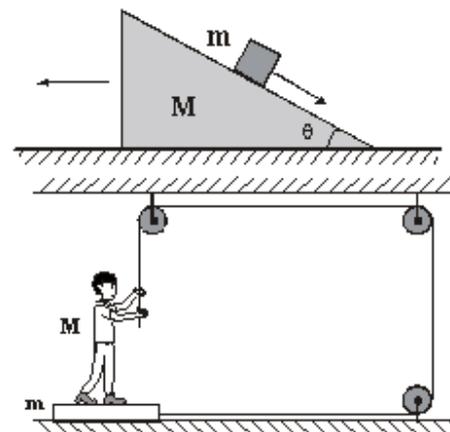
Q 5. The friction coeffficient between the board and the floor shown in figure is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor.
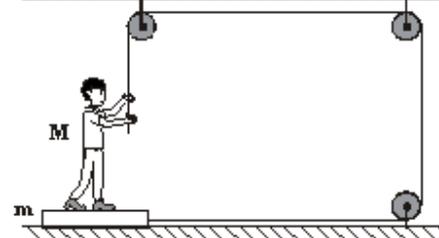
Q 6. Block A weights 4 N and block B weights 8 N. The coefficient of kinetic friction is 0.25 for all surfaces. Find the force F to slide B at a constant speed when (a) A rests on B and moves with it (b) A is held at rest and (c) A and B are connected by a light cord passing over a smooth pulloy as shown in Fig. (A), (B) and (C) respectively.
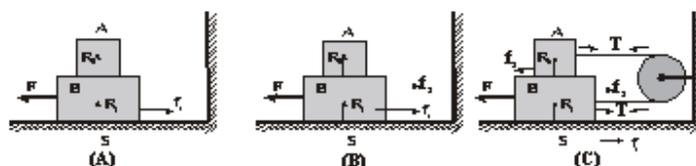
Q 7. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that it neither slips down nor skids up ?

Q 8. A hemispherical bowl of radius R is rotated about its axis of symmetry whcih is kept vertical. A small block is kept in the bowl at a position where the radius makes an angle $\theta$ with the vertical. The block rotates with the bowl without any slipping. The friction coefficient between the block and the bowl surface is $\mu$. Find the range of the angular speed for which the block will not slip.

Q 9.    A block of mass 2 kg is pushed against a rough vertical wall with a force of 40 N, coefficient of static friction being 0.5. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move ? If yes, in which direction? If no, find the frictional force exerted by the wall on the block.

Q 10.   Consider the situation shown in figure the block B moves on a frictionless surface, while the coefficient of friction between A and the surface on which it moves is 0.2. Find the acceleration with which the masses move and also the tension in the strings. Take g = 10 m/s².
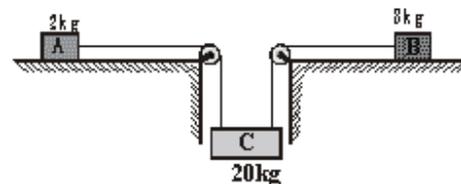
Q 11.   Find the acceleration of masses $m_1$ and $m_2$. The string and the pulley are massless and frictionless.
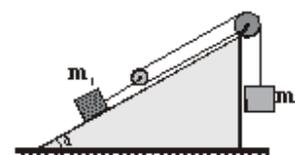
Q 12.   Two blocks of masses m = 5 kg and M =10 kg are connected by a string passing over a pulley B as shown. Another string connects the centre of pulley B to the floor and passes over another pulley A as shown. An upward force F is applied at the centre of pulley A. Both the pulleys are massless.
Find the acceleration of blocks m and M if F is :
(a) 100 N            (b) 300 N            (c)  500 N(take g = 10 m/s²)
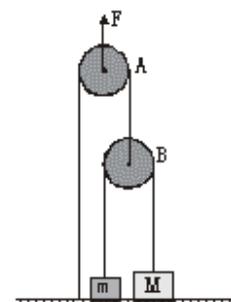
Q 13.   In the arrangement shown in fig. , the mass of body 1 is $\eta$ =4.0 times as great as that of body 2. The height h = 20 cm. The masses of the pulleys and the threads, as well as the friction, are negligible. At a certain moment body 2 is released and the arrangement set in motion. What is the maximum height that body 2 will go up to?
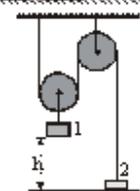
Q 14.   Find the acceleration of rod A and wedge B in the arrangement shown in fig. If the ratio of the mass of the wedge to that of the rod equals $\eta$, and the friction between all contact surfaces is negligible.
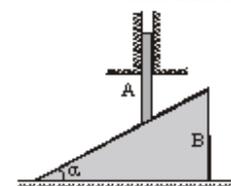
Q 15.   In the arrangement shown in Fig. The mass of ball 1 is $\eta$ = 1.8 times as great as that of rod 2. The length of the latter is $l$ = 100 cm. The masses of the pulleys and the threads, as well as the friction, are negligible. The ball is set on the same level as the lower end of the rod and then released. How soon will the ball be opposite the upper end of the rod?
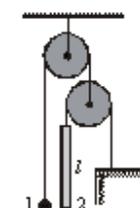
Q 16.   At the moment t = 0 the force F = at is applied to a small body of mass m resting on a smooth horizontal plane (a is a constant). The permanent direction of this force forms an angle $\alpha$ with the horizontal. Find:
(a) the velocity of the body at the moment of its breaking off the plane;
(b) the distance traversed by the body up to this moment.
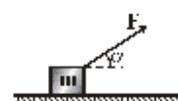
Q 17. A toy truck of mass M is moving towards left with an acceleration $a_1$ as shown in figure. It is connected to a mass $m_1$ with a massless and frictionless string, passing over a movable massless pulley, to which another mass $m_2$ is connected. Find the force acting on the truck towards right and the accelerations of masses $m_1$ and $m_2$, if F = 20 N, M = 2 kg, $m_1$ = 1 kg, $m_2$ = 0.5kg and g = 10 m/s².
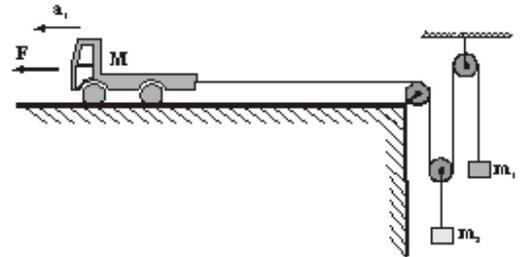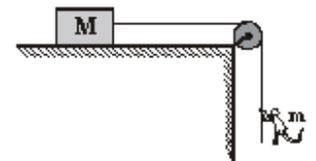
Q 18. Calculate the height upto which an insect can crawl up a fixed bowl in the form of a hemisphere of radius r. Given, coefficient of friction is $1/\sqrt{3}$.

Q 19. A painter of mass M stand on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown. He pulls each rope with force F and moves upward with a uniform acceleration 'a'. Find 'a' neglecting the fact that no one could do this for long time.
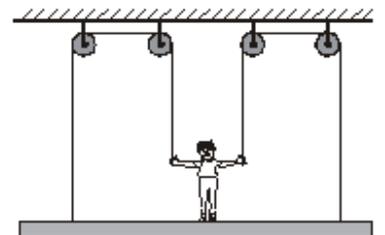
Q 20. A monkey of mass m clings to a rope slung over a fixed pulley. The opposite end of the rope is tied to a weight of mass M lying on a horizontal plate. The coefficient of friction between the weight and the plate is $\mu$. Find the acceleration of both bodies and the tension of the rope for three cases.
(a) the monkey dose not move with respect to the rope.
(b) the monkey moves upwards with respect to the rope with an acceleration b.
(c) the monkey moves downwards with respect to the rope with an acceleration b.

Q 21. For the pulley system shown, each of the cables at A and B is given a velocity of 2 m/s in the direction of the arrow. Determine the upward velocity $v$ of the load m.
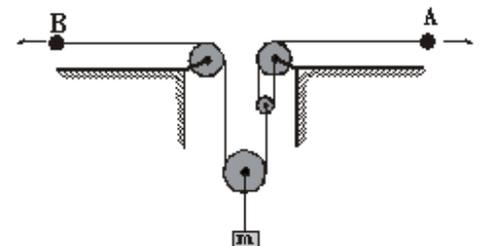
Q 22. In the system shown in figure $m_A = 4m$, $m_B = 3m$ and $m_C = 8m$. Friction is absent everywhere. String is light and inextensible. If the system is released from rest find the acceleration of each block.
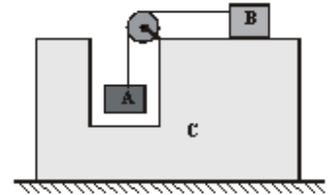
Q 23. In arrangement shown in the figure, find the acceleration of M. The pulleys are light. The strings are massless.
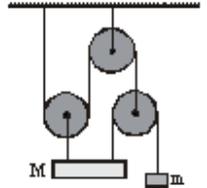
Q 24. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity $\omega$ in a circular path of radius R (figure). A smooth groove AB of length $L(<<R)$ is made on the surface of the table. The groove makes an angle $\theta$ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.
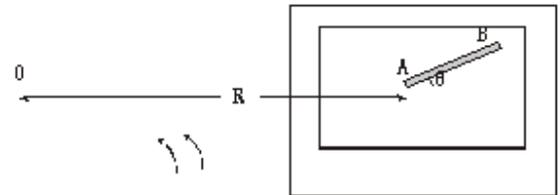
Q 25. A block of mass m slips on a rough horizontal table under the action of a horizontal force applied to it. The coefficient of friction between the block and the table is $\mu$. The table does not move on the floor. Find the total frictional force applied by the floor on the legs of the table. Do you need the friction coefficient between the table and the floor or the mass of the table ?

## Answers

Q 1.  $\frac{19}{29}$ g(up),  $\frac{17}{29}$ g (down),  $\frac{21}{29}$ g (down), 0.25 s

Q 2.  15 kg, 1800 N

Q 3.  $\frac{M'+m}{\cot\theta-1}$

Q 4.  $a=\frac{(M+m)g\sin\theta}{M+m\sin^2\theta}$

Q 5.  $\frac{\mu(M+m)g}{1+\mu}$

Q 6.  (a) 3 N, (b) 4 N, (c) 5 N

Q 7.  Between 14.7 km/h and 54 km/hr

Q 8.  $\left[\frac{g(\sin\theta-\mu\cos\theta)}{R\sin\theta(\cos\theta+\mu\sin\theta)}\right]^{\frac{1}{2}}$

Q 9.  it will move at an angle of $53^0$ with the 15 N force

Q 10.  $T_1 = 32$ N ,  $T_2 = 48$ N

Q 11.  $a = \frac{m_1\sin\alpha - 2m_2}{m_1+4m_2}g$  and $2a = 2\left(\frac{m_1\sin\alpha - 2m_2}{m_1+4m_2}\right)g$

Q 12.  (a) Blocks will remain stationary a = 0, (b) 5 m/s² from and zero for M., (c) Acceleration of m $a_1$= 15 m/s², Acceleration of M  $a_2$ = 2.5 m/s².

Q 13.  H = 6hη/(η + 4) = 0.6 m.

Q 14.  $w_A = g/(1 + \eta \cot^2\alpha)$, $w_B = g/(\tan\alpha + \eta \cot \alpha)$

Q 15.  $\sqrt{2l(4+\eta)/3g(2-\eta)} = 1.4\ s$.

Q 16.  $(a)\ v = \dfrac{mg^2 \cos \alpha}{2a \sin^2 \alpha}$, $(b)\ s = \dfrac{m^2 g^3 \cos \alpha}{6a^2 \sin^3 \alpha}$

Q 17.  T = force acting on truck towards right = 4.21 N; $a_3$ = acceleration of $m_1$ = 5.79 m/s² (downwards); $a_2$ = acceleration of $m_2$ = 6.84 m/s² (upwards)

Q 18.  $h = r\left(1 - \dfrac{\sqrt{3}}{2}\right)$

Q 19.  $\dfrac{4F}{M+m} - g$

Q 20.  $(a)\ a_1 = \dfrac{(m - \mu M)}{m + M}\,g$, $T = \dfrac{mM[g(1+\mu)}{m + M}$  $(b)\ a_2 = \dfrac{m[g+b] - \mu Mg}{m + M}$, $T = \dfrac{mM[g(1+\mu)+b]}{m + M}$  $(c)\ a_3 =$

$\dfrac{m(g-b) - \mu Mg}{m + M}$  $T = \dfrac{mM[g(1+\mu)+b]}{m + M}$

Q 21.  1.5 m/s

Q 22.  $a_1 + a_2 = \dfrac{5}{8}g$

Q 23.  $a = \left(\dfrac{M + 5m}{M + 25m}\right)g$

Q 24.  $\sqrt{\dfrac{2L}{\omega^2 R \cos \theta}}$