Consider a raindrop falling through a cloud of tiny water droplets. Some of these tiny droplets get attached to this raindrop as it falls increasing its mass. Assume constant growth in the mass of the raindrop as a function of distance x of fall. Find out the acceleration of the raindrop assuming it to be constant.

Solution

Let m be the mass of the raindrop at any instant.

$F_{ext} = \frac{d}{dt}(mv)$ where v is instantaneous velocity of the raindrop.

$F_{ext} = mg = m\frac{dv}{dt} + v\frac{dm}{dt}$

Given, $m = m_0 + kx$ where m_0 is the initial mass and k is some constant.

So, $(m_0 + kx)g = (m_0 + kx)\frac{dv}{dt} + v\frac{d(m_0 + kx)}{dt}$

$\Rightarrow (m_0 + kx)g = (m_0 + kx)a + kv^2$

On differentiating with respect to t,

$\frac{d(m_0 g)}{dt} + kvg = \frac{d(m_0 a)}{dt} + kx\frac{da}{dt} + kva + 2kva$

$\Rightarrow 0 + kvg = 0 + 0 + 3kva (\because m_0, a = \text{constant})$

$\therefore a = \frac{g}{3}$