The largest value of the nonnegative integer a for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$

is

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Solution

We have,

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{-a(x-1)+\sin(x-1)}{x-1+\sin(x-1)}\right\}^{1+\sqrt{x}}=\frac{1}{4}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{-a+\dfrac{\sin(x-1)}{(x-1)}}{1+\dfrac{\sin(x-1)}{(x-1)}}\right\}^{1+\sqrt{x}}=\frac{1}{4}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{-a+\dfrac{\sin(x-1)}{(x-1)}}{1+\dfrac{\sin(x-1)}{(x-1)}}\right\}^{\lim\limits_{x\to 1}(1+\sqrt{x})}=\frac{1}{4}$$

$$\Rightarrow \left[\frac{\lim\limits_{x\to 1}\left\{-a+\dfrac{\sin(x-1)}{(x-1)}\right\}}{\lim\limits_{x\to 1}\left\{1+\dfrac{\sin(x-1)}{(x-1)}\right\}}\right]^2=\frac{1}{4}$$

$$\Rightarrow \left[\frac{\lim\limits_{x\to 1}(-a)+\lim\limits_{x\to 1}\dfrac{\sin(x-1)}{(x-1)}}{\lim\limits_{x\to 1}1+\lim\limits_{x\to 1}\dfrac{\sin(x-1)}{(x-1)}}\right]^2=\frac{1}{4}$$

$$\Rightarrow \left[\frac{-a+1}{1+1}\right]^2=\frac{1}{4}$$

$$\Rightarrow (1-a)^2=1$$

$\Rightarrow a = 0, 2$

Hence, 2.