

The largest value of the nonnegative integer  $a$  for which

$$\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

is

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### Solution

We have,

$$\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

$$\Rightarrow \lim_{x \rightarrow 1} \left\{ \frac{-a(x-1) + \sin(x-1)}{x-1 + \sin(x-1)} \right\}^{1+\sqrt{x}} = \frac{1}{4}$$

$$\Rightarrow \lim_{x \rightarrow 1} \left\{ \frac{-a + \frac{\sin(x-1)}{(x-1)}}{1 + \frac{\sin(x-1)}{(x-1)}} \right\}^{1+\sqrt{x}} = \frac{1}{4}$$

$$\Rightarrow \lim_{x \rightarrow 1} \left\{ \frac{-a + \frac{\sin(x-1)}{(x-1)}}{1 + \frac{\sin(x-1)}{(x-1)}} \right\}^{\lim_{x \rightarrow 1} (1+\sqrt{x})} = \frac{1}{4}$$

$$\Rightarrow \left[ \frac{\lim_{x \rightarrow 1} \left\{ -a + \frac{\sin(x-1)}{(x-1)} \right\}}{\lim_{x \rightarrow 1} \left\{ 1 + \frac{\sin(x-1)}{(x-1)} \right\}} \right]^2 = \frac{1}{4}$$

$$\Rightarrow \left[ \frac{\lim_{x \rightarrow 1} (-a) + \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x-1)}}{\lim_{x \rightarrow 1} 1 + \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x-1)}} \right]^2 = \frac{1}{4}$$

$$\Rightarrow \left[ \frac{-a+1}{1+1} \right]^2 = \frac{1}{4}$$

$$\Rightarrow (1-a)^2 = 1$$

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$$\Rightarrow a = 0, 2$$

Hence, 2.