The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\, dt, x \in R$, which are parallel to the line y = 2x, are equal to:

(1) ± 1 (2) ± 2 (3) ± 3 (4) ± 4

[JEE Main 2013]

Solution

If x > 0, $y = \int_0^x t dt = \frac{x^2}{2}$

Slope of tangent = $\frac{dy}{dx} = x$

Since the tangent is parallel to the line y = 2x whose slope = 2,

$$\frac{dy}{dx} = x = 2$$

$$y = \frac{x^2}{2} = 2$$

So, the point of contact is (2, 2) and the equation of the tangent is,

y - 2 = 2(x – 2)

Or, y = 2x – 2 whose intercept on the x-axis is 1.

If x < 0, $y = \int_0^x -t dt = -\frac{x^2}{2}$

Slope of tangent = $\frac{dy}{dx} = -x$

Since the tangent is parallel to the line y = 2x whose slope = 2,

$\frac{dy}{dx} = -x = 2$ or x = -2

$$y = -\frac{x^2}{2} = -2$$

So, the point of contact is (-2, -2) and the equation of the tangent is,

y + 2 = 2(x + 2)

Or, y = 2x + 2 whose intercept on the x-axis is -1. Hence, option (1).