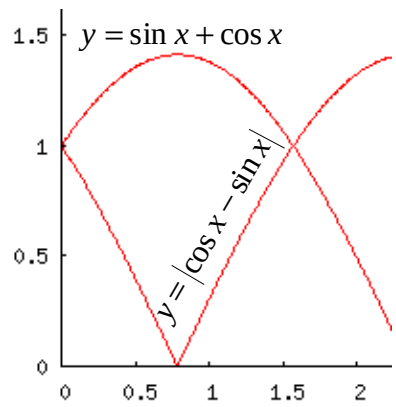
Only one option is correct.

The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

(A) $4(\sqrt{2}-1)$ (B) $2\sqrt{2}(\sqrt{2}-1)$ (C) $2(\sqrt{2}+1)$ (D) $2\sqrt{2}(\sqrt{2}+1)$

[JEE Advanced 2013]

Solution



The required area,

$$= \int_0^{\pi/2} \left[\sin x + \cos x - \left(|\cos x - \sin x|\right)\right] dx$$

$$= \int_0^{\pi/4} \left[\sin x + \cos x - (\cos x - \sin x)\right] dx + \int_{\pi/4}^{\pi/2} \left[\sin x + \cos x - (\sin x - \cos x)\right] dx$$

$$= \int_0^{\pi/4} 2\sin x dx + \int_{\pi/4}^{\pi/2} 2\cos x dx$$

$$= -2\cos x\Big|_0^{\pi/4} + 2\sin x\Big|_{\pi/4}^{\pi/2}$$

$$= -2\left(\frac{1}{\sqrt{2}} - 1\right) + 2\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= 4 - \frac{4}{\sqrt{2}} = 2\sqrt{2}\left(\sqrt{2} - 1\right)$$

Hence, (B).