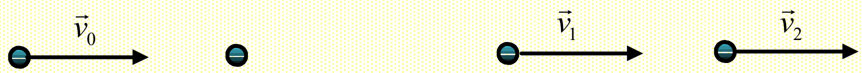
In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

(1) $\frac{v_0}{4}$ (2) $\sqrt{2}v_0$ (3) $\frac{v_0}{2}$ (4) $\frac{v_0}{\sqrt{2}}$

Solution

Note: Usually, the final kinetic energy is such problems is not more than the initial kinetic energy. In this problem, the increase in kinetic energy that is taking place must be coming from the internal energy.



$$K_1 = \frac{1}{2}mv_0^2$$

$$K_2 = \frac{3}{2}K_1 = \frac{3}{4}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\therefore v_1^2 + v_2^2 = \frac{3}{2}v_0^2$$

Conservation of linear momentum gives,

$mv_0 = mv_1 + mv_2$ Or, $v_0 = v_1 + v_2$

Relative velocity = $v_2 - v_1$

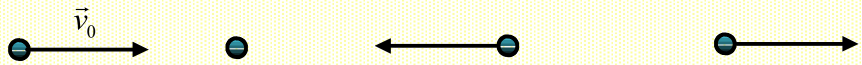
To find $v_2 - v_1$, let us first find $v_2 v_1$.

$$(v_2 + v_1)^2 = v_2^2 + v_1^2 + 2v_2v_1$$

$$\therefore v_0^2 = \frac{3}{2}v_0^2 + 2v_2v_1$$

$$\therefore v_2v_1 = -\frac{v_0^2}{4}$$

Note: Since, $\vec{v}_1$ and $\vec{v}_2$ have opposite signs, actually the moving particle bounces back after colliding with stationary particle.



Now, $(v_2 - v_1)^2 = v_2^{\ 2} + v_1^{\ 2} - 2v_2v_1$

$$\therefore (v_2 - v_1)^2 = \frac{3}{2}v_0^{\ 2} - 2\times\left(-\frac{v_0^{\ 2}}{4}\right)$$

$$\therefore (v_2 - v_1)^2 = \frac{3}{2}v_0^{\ 2} + \frac{v_0^{\ 2}}{2} = 2v_0^{\ 2}$$

$$\therefore v_2 - v_1 = \sqrt{2}v_0$$

Hence, Option (2).

Based on JEE Main 2018 - 123IITJEE