Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0 Ns^{-1}$ and $\beta = 1.0 N$. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}| = \frac{1}{3} Nm$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j}) ms^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6} m$

Solution

$$\vec{F} = (\alpha t\hat{i} + \beta\hat{j}) = (t\hat{i} + \hat{j}) = m\frac{d\vec{v}}{dt} = \frac{d\vec{v}}{dt}$$

$$\therefore \int_0^t (t\hat{i} + \hat{j})dt = \int_0^{\vec{v}} d\vec{v}$$

$\therefore \vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$, Or $\vec{v}_{t=1s} = \frac{1}{2}\hat{i} + \hat{j}$ [Option (C) is correct]

Again, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$

$$\therefore \int_0^{\vec{r}} d\vec{r} = \int_0^t \frac{t^2}{2}\hat{i} + t\hat{j}dt \quad \text{Or,} \quad \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$$

$\therefore \vec{r}_{t=1s} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$, Or $|\vec{r}_{t=1s}| = \sqrt{\frac{1}{6^2} + \frac{1}{2^2}} = \frac{\sqrt{10}}{6}$ m

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}\right) \times \left(t\hat{i} + \hat{j}\right) = \frac{t^3}{6}\hat{k} - \frac{t^3}{2}\hat{k} = -\frac{t^3}{3}\hat{k}$$

$\vec{\tau}_{1s} = -\frac{1}{3}\hat{k}$, Or $|\vec{\tau}_{1s}| = \frac{1}{3}$ Nm [Option (A) is correct]

Hence, Options (A) & (C). [Based on JEE Adv. 2018 - 123IITJEE]