The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = Kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

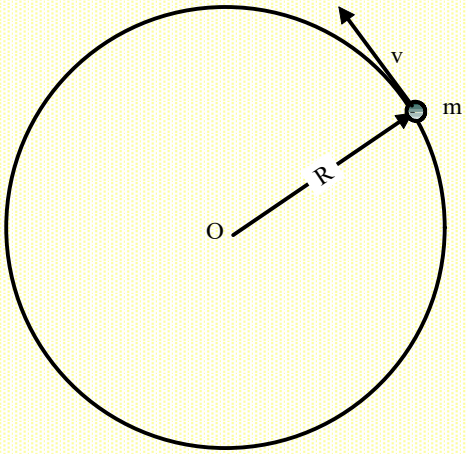
(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$

(C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\frac{mk}{2}}R^2$

Solution

$$F = -\frac{dV}{dr} = -\frac{d\left(kr^2/2\right)}{dr} = -kr$$

Let us now consider the circular motion.



$$\left|\vec{F}\right| = kR = \frac{mv^2}{R}$$

$$\therefore v = R\sqrt{\frac{k}{m}}$$

$$L = mvR = m\left(R\sqrt{\frac{k}{m}}\right)R = R^2\sqrt{mk}$$

Hence, Options (B) & (C). [Based on JEE Adv. 2018 - 123IITJEE]